Name: __________________________________________

Your answers must include all steps of your derivations, or points will be deducted.

This is closed book exam. You may not use electronic devices, notes, textbooks or other written materials.

Throughout the exam we use right-handed coordinate systems only.

Good luck!

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1. True or False (10 Points)

True or False: Circle one choice only for each part. (1 point each)

T / F: The canonical view volume preserves the relative ordering of the virtual objects z-values (depths).

T / F: This quaternion represents the identity rotation: \((w, x, y, z) = (1, 0, 0, 0)\).

T / F: The inverse of a 3D rotation matrix is simply its transpose.

T / F: The surface normal is the vector that is parallel to the surface.

T / F: Vector \((1, 1, 1)^T\) is a unit vector.

T / F: In rotations with Euler angles the order of rotations is always first about the x axis, then the y axis, then the z axis.

T / F: An affine transformation is any transformation that preserves collinearity and ratios of distances.

T / F: The law of associativity always applies to a sequence of affine transformations.

T / F: In the chain of transformations for rendering points on the screen, the last matrix applied \((D)\) is responsible for illumination models of the light sources in the virtual scene.

T / F: The light rays emanating from a perfect point light source are never parallel, unless lenses or mirrors are involved.
2. Vector Properties (10 Points)

a) Given two vectors \( \mathbf{a} \) and \( \mathbf{b} \) in three-dimensional space, how do you calculate the angle between these vectors? (3 points)

b) Given \( |\mathbf{a}| = |\mathbf{b}| = 1 \), and \( \mathbf{a} \) perpendicular to \( \mathbf{b} \), provide a third vector \( \mathbf{c} \) such that \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) is a rotation of the standard coordinate axes \( x, y, z \). (3 points)

c) How do two vectors \( \mathbf{v} \) and \( \mathbf{w} \) have to be oriented to minimize the dot product? (2 points)

d) How do two vectors \( \mathbf{v} \) and \( \mathbf{w} \) have to be oriented to minimize the magnitude of the cross product? (2 points)
3. Camera Matrix (10 Points)

Suppose the following matrix product is used as the camera matrix:

\[
C = \begin{bmatrix}
    a & b & c & 0 \\
    d & e & f & 0 \\
    g & h & i & 0 \\
    0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
    1 & 0 & 0 & j \\
    0 & 1 & 0 & k \\
    0 & 0 & 1 & l \\
    0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

You may assume that it is correctly formed. Determine the following in terms of parameters a, b, c, ..., l in world X, Y, Z coordinates.

a) The camera position \( p \). (2 points)

b) The camera up vector \( u \). (2 points)

c) The camera direction vector \( d \) (=direction the camera is facing). (2 points)

d) The inverse camera matrix \( C^{-1} \). (A single 4x4 matrix is expected.) (4 points)
4. Illumination Models (10 Points)

Consider the Phong illumination model as discussed in class, with one object with diffuse coefficients $k_d$ and specular coefficients $k_s$.

$$c = \sum_i c_{li} (k_d(L_i \cdot n) + k_s(R \cdot e)^p + k_a)$$

Assume there is no ambient lighting for this problem.

a) Given a fixed set of lights and $k_d \neq k_s$, when will the appearance of the object change? (3 points)

b) Describe the appearance of the object if $k_s = [0, 0, 0]^T$ and $k_d \neq k_s$, given a directional light with no attenuation. (2 points)

c) Consider a slightly simpler formulation of the Phong illumination model with a material color parameter $C$, scalar diffuse coefficient $d$, and scalar specular coefficient $s$. Assume that $d$ and $s$ are in the range $[0, 1]$. Give a formula for $k_d$ and $k_s$ in terms of $C$, $d$, and $s$. (Choose a maximally expressive model under the constraint that our parameter names are accurate and logical.) (3 points)

d) The new model described above, while it is more realistic for many materials, has one less degree of freedom. Give an example of a pair of material coefficients ($k_d, k_s$) which cannot be expressed in the new model. (2 points)
5. Lighting (10 Points)

a) What is the difference between the concepts of Global Illumination and Local Illumination? (4 points)

a) Explain the difference between faceted and smooth shading, and explain how both the Gouraud and Phong shading methods work, and their limitations. (4 points)

b) Explain the difference between the terms Phong shading and Phong illumination model. (2 points)
6. Shaders (10 Points)

a) Give an example of a GLSL vertex shader and a fragment shader. (You are not responsible for GLSL syntactic details, but you should have a general idea of how GLSL shaders work and what they look like.) (6 points)

b) Describe how a GLSL shader fits into the OpenGL pipeline, including what general steps must be accomplished by the application program in order to use a shader program. (4 points)
7. Texture Mapping (10 Points)

A 512 × 256 texture image is mapped onto a 3D parallelogram defined by vertices $P_{00}, P_{10}, P_{01}, P_{11}$, as shown below.

Pixel (0, 0) in the texture maps to $P_{00}$. Pixel (511, 0) maps to $P_{10}$, etc.

What is the mapping from pixel (s, t) in the texture image to 3D point $P$ on the quad? Develop the math for point $P$ as a function of $s$ and $t$. 

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![Diagram of a 3D parallelogram with vertices labeled $P_{00}, P_{01}, P_{10}, P_{11}$]
8. Scene Graph (10 Points)

The figure above illustrates a mini solar system. Given the partial source code below, complete the rest for this solar system. Use the function addChild(____). For example, Translation1.addChild(Rotation1);

Sun = new Sun();
Moon = new Moon();
World = new Transform(...);
PlanetA = new Planet('A');
PlanetB = new Planet('B');
Rotation1 = new Transform(...); Translation1 = new Transform(...);
Rotation2 = new Transform(...); Translation2 = new Transform(...);
Rotation3 = new Transform(...); Translation3 = new Transform(...);