

University of California San Diego
CSE167: Introduction to Computer Graphics
Final Examination
Thursday December 11, 2008

Instructor: Jürgen P. Schulze

Name: _____

Please write your name or initials at the top of every page.

Please include all steps of your derivations in your answers to show your understanding of the problem. Try not to write more than the recommended amount of text. If your answer is a mix of correct and substantially wrong arguments, we will consider deducting points for incorrect statements. You may not use calculators, cell phones, notes, textbooks or other materials during this exam. There are twelve questions for a total score of 100 points.

Good luck!

1. Points (4 points)

Given four homogeneous points $p_0 = (9, 12, 6, 1.5)$, $p_1 = (12, 16, 8, 2)$, $p_2 = (9, 12, 6, 1)$, and $p_3 = (18, 24, 12, 3)$. All of them except one represent the same 3D point.

- a) What is that 3D point? (3 points)
- b) Which of the homogeneous points p_0, \dots, p_3 represents a different 3D point? (1 point)

Name or initials: _____

2. Matrices (5 points)

Derive the inverse of each of the following three matrices (1 + 1 + 3 points)

$$M_0 = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; M_1 = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix}; M_2 = \begin{bmatrix} a & 0 & 0 & d \\ 0 & b & 0 & e \\ 0 & 0 & c & f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Name or initials: _____

3. Parametric Object (8 points)

Given a cone that has its apex at $(0, 0, 3)$, and whose intersection with the xy -plane is a unit circle.

a) Write a parametric equation of the form $\mathbf{p}(u, v)$ to describe the surface of this cone. Remember that $\mathbf{p}(u, v)$ consists of three functions $x(u, v)$, $y(u, v)$ and $z(u, v)$. The curve $\mathbf{p}(u, 0)$, $u \in [0, 1]$ should map to a unit circle in the xy -plane, and $\mathbf{p}(u, 1)$, $u \in [0, 1]$ should be the apex. (4 points)

b) Derive equations for two tangent vectors and the normal at any point (u, v) . You do not need to normalize the normal vector to unit length. (4 points)

Name or initials:

4. Texture Mapping (8 points)

a) What artifacts might occur when a planar surface texture mapped with a 256x256 pixel checkerboard pattern is rendered far away so that it covers roughly a 32x32 pixel window? Name two techniques that can be used to minimize these problems. (4 points)

b) Name two fundamental differences between bump mapping and displacement mapping. (4 points)

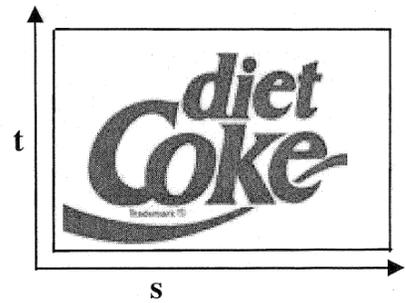
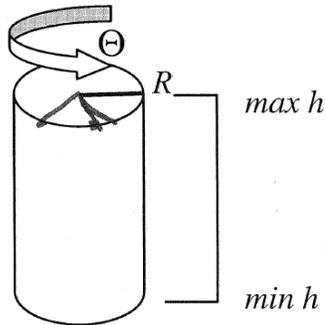
Name or initials: _____

5. Texture Coordinates (6 points)

You are given the task of putting the diet Coke label on the 2 liter bottle as shown (it only goes 25% around the bottle and is positioned as shown). The bottle has two parts: a cylindrical bottom and a conical top. The bottom of the label should be half-way up the cylindrical part of the bottle, and its height should be 25% of the height of the cylindrical part of the bottle. What are the texture coordinates which perform this operation? (In other words: express s and t as functions of $min\ h$, $max\ h$, and θ)

Be sure to describe any assumptions you make. Although the texture is not square, you can assume the range in s and t is $[0..1]$.

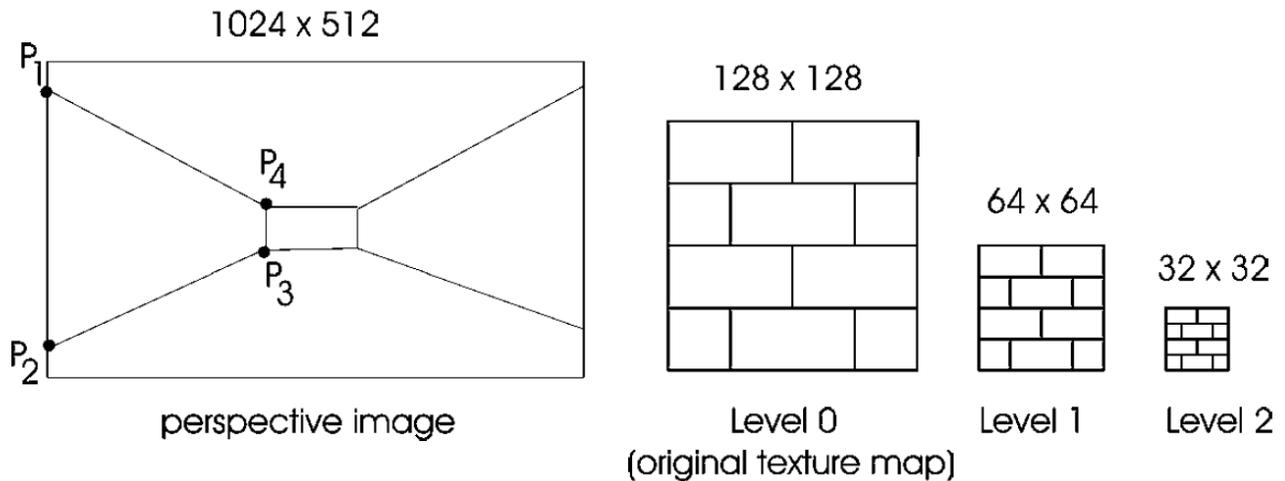
$$\begin{aligned}x &= R \cos(\Theta) \\y &= R \sin(\Theta) \\z &= h\end{aligned}$$



Name or initials: _____

6. Mip-Mapping (6 points)

Suppose we have a brick wall that forms the left-hand wall of a corridor in a maze game, as shown in the image below, and it is defined (in world coordinates) by points P_1 , P_2 , P_3 , P_4 . Assume that the brick wall is to be 16 bricks high and 200 bricks long.



a) Using the height of the brick wall as seen in the image, estimate (with derivation) how many texels a screen pixel covers, both for near points on the wall, i.e., on the edge P_1P_2 , and at distant points on the wall, i.e., on the edge P_3P_4 . (3 points)

b) On the perspective image, sketch approximately what regions of the wall will use each of the levels of the Mip-Map image pyramid on the right. (3 points)

Name or initials:

7. Shaders (9 points)

a) Write down pseudo-code for a vertex and fragment shader that perform per-pixel diffuse shading for a single light source. The input to the vertex shader is the vertex position, normal, and diffuse color, and the light position and color. You can use your own names for these variables, there is no need to use the OpenGL names. (5 points)

As a reminder, the diffuse shading equation is:

$$c_d = c_l k_d (n \cdot L)$$

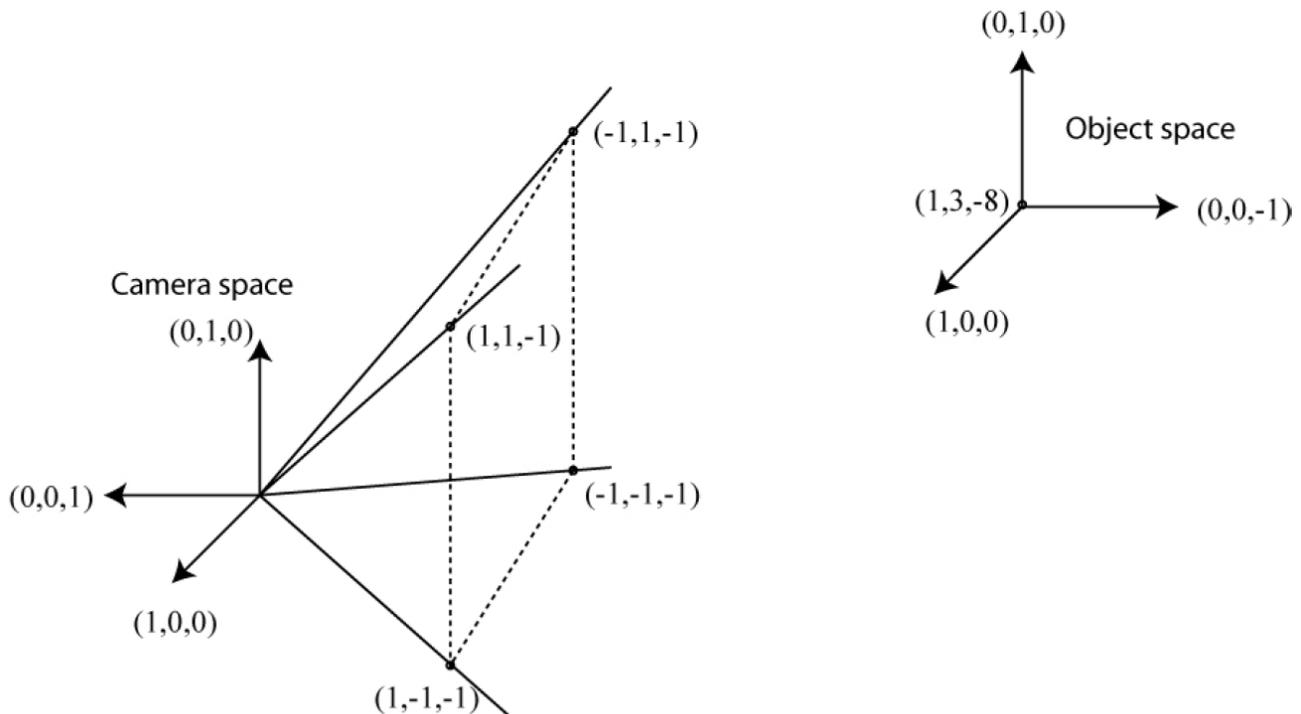
b) List one advantage and one disadvantage of computing shading on a per-pixel basis compared to shading on a per-vertex basis. (4 points)

Name or initials: _____

8. View Frustum Culling (10 points)

Given the perspective view frustum shown in the figure below. The top bounding plane of the view frustum is determined by the plane going through the points $(0, 0, 0)$, $(1, 1, -1)$, and $(-1, 1, -1)$ in camera coordinates. Note that the other bounding planes will not be relevant to this problem. In addition, there is an object coordinate system defined by basis vectors $(0, 1, 0)$, $(1, 0, 0)$, $(0, 0, -1)$ and the origin $(1, 3, -8)$ in camera coordinates. Note that the order of the basis vectors matters!

Assume there is an object with a bounding sphere with radius 2 centered at $(8, 1, 1)$ in object coordinates. Determine if this bounding sphere intersects with the top bounding plane of the view frustum. You should do this by transforming the center of the bounding sphere from object to camera coordinates. Then you need to compute the distance from the bounding sphere center in camera coordinates to the top bounding plane.



Name or initials:

9. Spline Approximation to an Arc (8 points)

Very efficient algorithms exist to draw cubic Bezier curves. In fact, these algorithms are so efficient that other types of curves are often converted to Bezier curves. For example, it is possible to approximate a 90 degree circular arc with a Bezier curve.

Recall that a cubic Bezier curve has four control points. Derive the coordinates of the four control points of a Bezier curve that approximates this circular arc. This approximation should touch and be tangent to the arc at both endpoints as well as the midpoint. Hint: Find control points such that the midpoint of the Bezier curve ($t=1/2$) lies on the midpoint of the circular arc.

Name or initials: _____

10. Surface Patch (10 points)

A bilinear surface patch is specified by four control points $p_{(0,0)} = (0, 0, 0)$, $p_{(1,0)} = (6, 0, 6)$, $p_{(0,1)} = (2, 4, 4)$, and $p_{(1,1)} = (5, 4, 4)$.

- Write down an equation of the form $x(u, v)$ that computes a point x on the surface patch given parameter values $0 < u, v < 1$. (5 points)
- Compute two tangent vectors and the unit surface normal at $(u, v) = (\frac{2}{3}, \frac{1}{2})$. Evaluate these vectors numerically. (5 points)

Name or initials:

11. Shadow Mapping (8 points)

- a) Describe the shadow mapping algorithm using a sketch and a few explanatory sentences. (4 points)
- b) List two potential problems or artifacts that may appear with shadow mapping, and explain what can be done to fix them. (2 points)
- c) List one advantage and one disadvantage of shadow mapping compared to shadow volumes. (2 points)

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12. L-Systems (10 points)

Given the following L-System grammar:

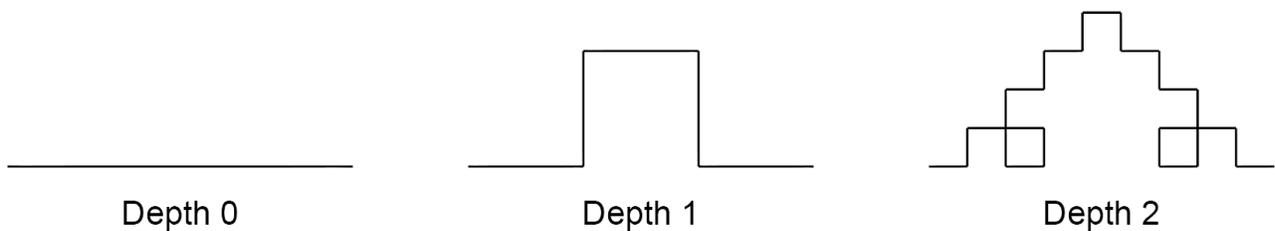
- #Z - rotate around the Z-axis by # degrees
- [- push the current position and angle onto the stack
-] - pop the stack
- A,B - draw a segment or execute a replacement rule

a) Given an L-System with axiom B, and the replacement rule

$B \leftarrow A 45Z A [-45Z B] [45Z B]$

Draw the output of the L-System for rendering depth of 0 (no replacements) 1, and 2. (5 points)

b) The picture below depicts an L-System with only one segment ID, A, and a single replacement rule. The L-System has been rendered at three depths (not to scale):



What might be a plausible axiom and replacement rule for this L-System? (5 points)

Name or initials:

13. CUDA (8 points)

a) What is the purpose of CUDA? (2 points)

b) Name three fundamental differences between CUDA and GLSL, and explain in one sentence for each of them. (6 points)