University of California San Diego
Department of Computer Science
CSE167: Introduction to Computer Graphics
Midterm Examination \#2
Tuesday, November $23^{\text {rd }}, 2010$
Instructor: Dr. Jürgen P. Schulze

Name: $\qquad$
Please write your name or initials at the top of every page before beginning the exam.
Please include all steps of your derivations in your answers to show your understanding of the problem. Try not to write more than the recommended amount of text. If your answer is a mix of correct and substantially wrong arguments we will consider deducting points for incorrect statements. You may not use calculators, notes, textbooks or other materials during this exam, except for one double sided, hand-written $3 x 5$ inch index card. There are eight questions for a total score of 100 points.

Good luck!

This space is for grading

| Exercise | Points |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| Total |  |

## 1) Linear Algebra (10 points)

1. In the following, assume we are working in 2D and that $S(a, b)$ scales by $a$ in $x, b$ in $y$; that $R(a)$ rotates by the angle a [in degrees], and $T(t, s)$ translates $t$ in $x$ and $s$ in $y$.
a) Which of the following pairs of transforms can be reversed without effecting the result (that is, applied in reverse order)? Write Yes if they can be reversed and No if they cannot. (5 points)
$\mathrm{T}(1,2) \mathrm{T}(2,3)$
$S(2,1) R(45)$
$R(45) T(1,1)$
$R(10) R(20)$
$S(2,2) R(45)$
b) Write down a sequence of transformations that has the effect of rotating an object by 45 degrees about the point (1,1). (5 points)

## 2) Environment Mapping (14 Points)

Given a cubic environment map and a light direction vector ( $\mathbf{x}, \mathbf{y}, \mathbf{z}$ ) on the surface of an object which uses this environment map for pure specular reflection (mirror-like), you need to figure out the color of the pixel to draw on the screen.
a) The light direction vector can be viewed as a ray intersecting the cubic environment map. How can you calculate which face the ray intersects? (3 points)
b) What are the coordinates of the intersection within that face? (3 points)
c) How do you get the ( $\mathbf{u}, \mathbf{v}$ ) coordinates for the environment map texture associated with this face? (2 points)

Now we add another object to the scene, this time one with a diffuse surface. We still want to use the same cubic environment map for shading.
d) What is the fundamental difference between the shading algorithm for the diffuse surface and that for specular surfaces? Explain your answer. (4 points)
e) What is the technical term for the environment map needed for diffuse shading? (2 points)

## 3) Joining Curves (12 Points)

Equations of two curves $Q$ and $R$ are given as follows:
$Q(t)=\left(1+t^{2}\right) P_{1}+\left(5 t-5 t^{2}\right) P_{2}+t^{2} P_{3}$.
$R(t)=\left(1+t^{2}\right) P_{3}+\left(5 t-5 t^{2}\right) P_{4}+t^{2} P_{5}$.
For both the curves $0<=\mathrm{t}<=1 . \mathrm{P}_{\mathrm{n}}$ are the control points.
a) Are both the curves $\mathrm{C}_{0}$ continuous at $\mathrm{Q}(1)$ and $\mathrm{R}(0)$ ? Why or why not? (6 points)
b) Are they both $C_{1}$ continuous at $Q(1)$ and $R(0)$ ? Why or why not? (6 points)

## 4) Bezier Curve (14 Points)

The most widely used curve in computer graphics is the Bezier curve. The quadratic Bezier curve is given by 3 points, $P_{0}, P_{1}$, and $P_{2}$. The cubic Bezier curve is given by 4 points, $P_{0}, P_{1}$, $\mathrm{P}_{2}$ and $\mathrm{P}_{3}$.

To draw another curve, we need to convert it to graphics primitives. Normally we draw a curve by drawing a set of line segments. However, it is possible to convert a curve to a Bezier curve. The advantage of this approach is that it takes many fewer Bezier curves than line segments to closely approximate our curve.

In this problem, we want to draw the parabola $\mathrm{y}=\mathrm{x} 2$.


We want to draw a section of the parabola from $(0,0)$ to $(1,1)$.
a) Determine the positions of the 3 controls points $\mathrm{P}_{0}, \mathrm{P}_{1}$, and $\mathrm{P}_{2}$ of the quadratic Bezier curve. Position these points so that the Bezier curve goes through $(0,0)$ and $(1,1)$ and is tangent to the parabola at these two points. (7 points)
b) Calculate the positions of the 4 controls points $\mathrm{P}_{0}, \mathrm{P}_{1}, \mathrm{P}_{2}$ and $\mathrm{P}_{3}$ of the cubic Bezier curve. Position these points so that the Bezier curve goes through $(0,0)$ and $(1,1)$ and is tangent to the parabola at these points. Also require that the point $(1 / 2,1 / 4)$ on the parabola be on the Bezier curve. That is, $P(1 / 2)=(1 / 2,1 / 4)$. ( 7 points)

## 5) Bilinear Surface Patch (12 Points)

A bilinear patch $x(u, v)$ is given by four control points $P_{0}=(2,0,1), P_{1}=(4,2,1), P_{2}=(2,2$, 0 ), and $P_{3}=(7,4,5)$; and $x(0,0)=P_{0}, x(1,0)=P_{1}, x(0,1)=P_{2}$, and $x(1,1)=P_{3}$.
a) Evaluate the patch at $(u, v)=(2 / 10,5 / 10)$. ( 6 points )
b) Compute the tangent vectors and the normal at the point from part a). (6 points)

## 6) Toon Shading (12 points)

a) What is the goal of toon shading? Explain. (2 points)
b) How can toon shading be accomplished in real-time? (2 points)
c) Explain how silhouette edges can be detected for toon shading. (4 points)
d) Name two parameters which the programmer can tweak the toon shading algorithm with.
(4 points)

## 7) Shadow Mapping (12 Points)

a) Describe the shadow mapping algorithm using a sketch and a few explanatory sentences. (8 points)
b) List two potential problems or artifacts that may appear with shadow mapping. (4 points)

## 8) L-System (14 Points)

The following L-system describes the Dragon curve. The L-system has the following parameters:

Varibles: X Y
Constants: F + -
Start string: $F X$
Rules: $(X \rightarrow X+Y F),(Y \rightarrow F X-Y)$
Here, F means "draw forward", - means "turn left $90^{\circ}$ ", and + means "turn right $90^{\circ}$ ". X and Y do not correspond to any drawing action and are only used to control the evolution of the curve.

Hint: The first level of recursion is: $F X+Y F$.
a) Generate the strings for the second and third level of recursion (6 points).
b) Draw this curve for the second and third level of recursion (8 points).

