CSE 167: Introduction to Computer Graphics Lecture #4: Projection

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Announcements

- Project 2 due Friday at 1pm
  - Grading starts at 12 noon
- Project 3 discussion Monday at 4pm

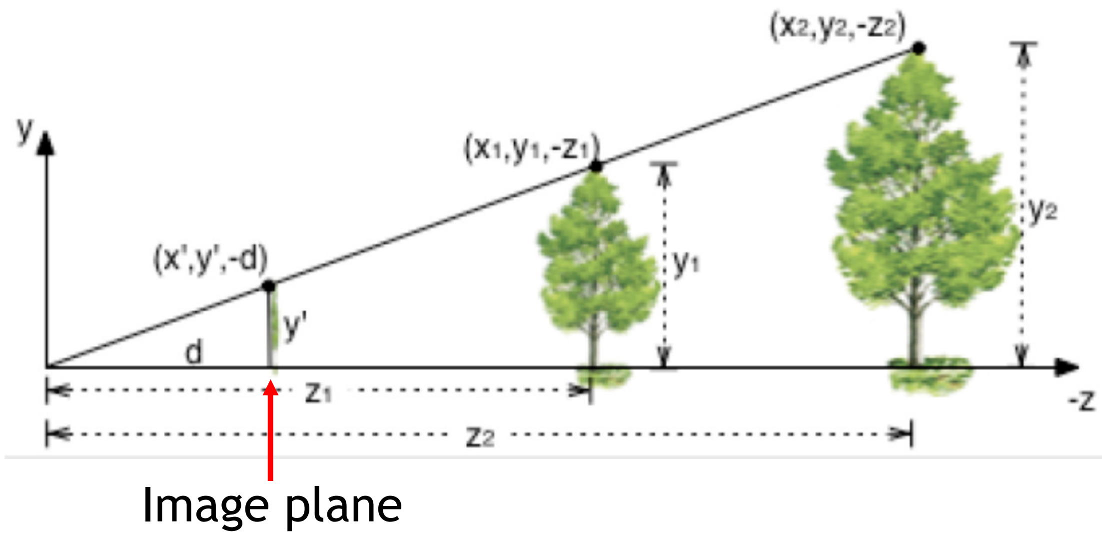
Perspective Projection

From law of ratios in similar triangles follows:

$$\frac{y'}{d} = \frac{y_1}{z_1} \rightarrow y' = \frac{y_1 d}{z_1}$$

Similarly: $$x' = \frac{x_1 d}{z_1}$$

By definition: $$z' = d$$



- We can express this using homogeneous coordinates and 4x4 matrices as follows

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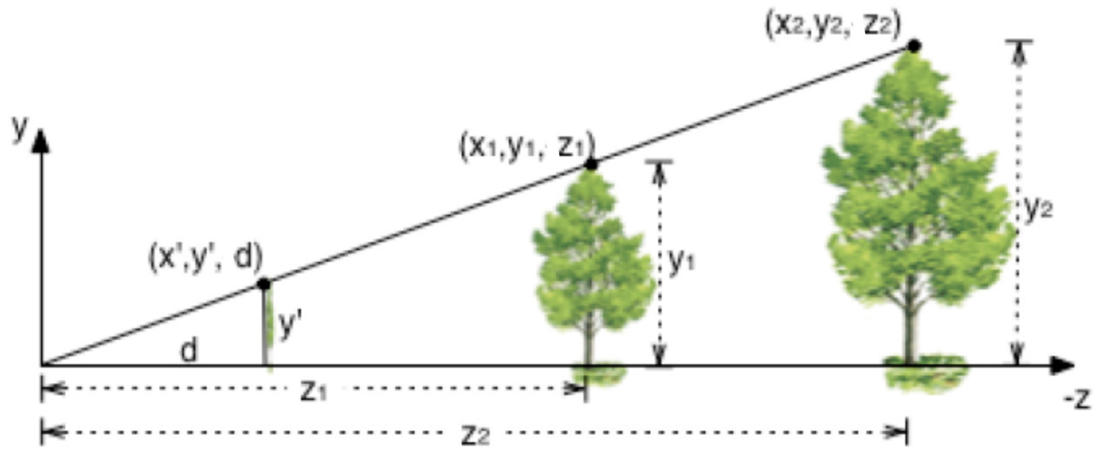
Perspective Projection

$$x' = \frac{x_1 d}{z_1}$$

$$y' = \frac{y_1 d}{z_1}$$

$$z' = d$$



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \Rightarrow \begin{bmatrix} xd/z \\ yd/z \\ d \\ 1 \end{bmatrix}$$

Projection matrix Homogeneous division

Perspective Projection

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} xd/z \\ yd/z \\ d \\ 1 \end{bmatrix}$$

Projection matrix P

- Using projection matrix, homogeneous division seems more complicated than just multiplying all coordinates by d/z, so why do it?
- It will allow us to:
  - Handle different types of projections in a unified way
  - Define arbitrary view volumes

Lecture Overview

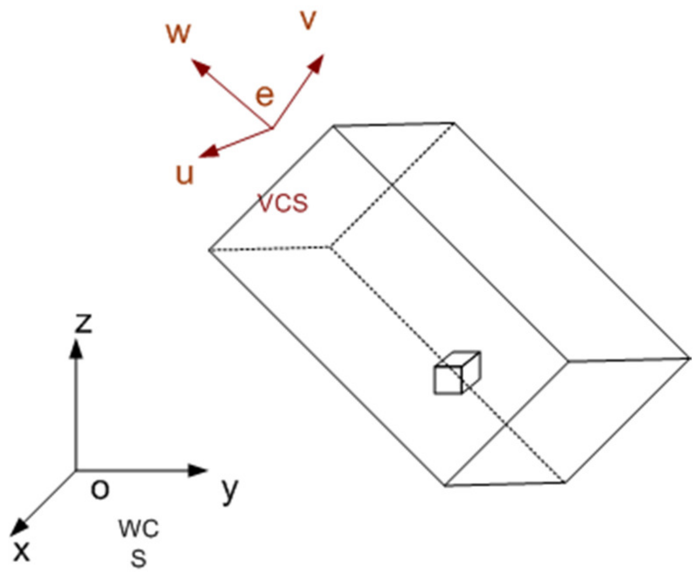
- View Volumes
- Vertex Transformation
- Rendering Pipeline
- Culling

View Volumes

- View volume = 3D volume seen by camera

Orthographic view volume

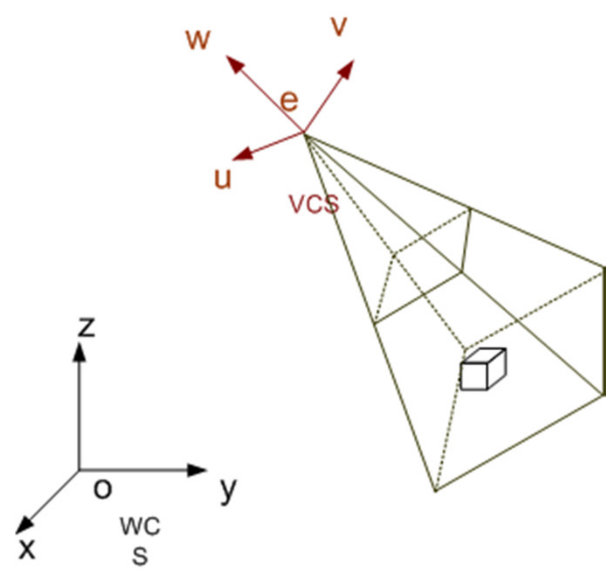
Camera coordinates



World coordinates

Perspective view volume

Camera coordinates



World coordinates

Projection Matrix

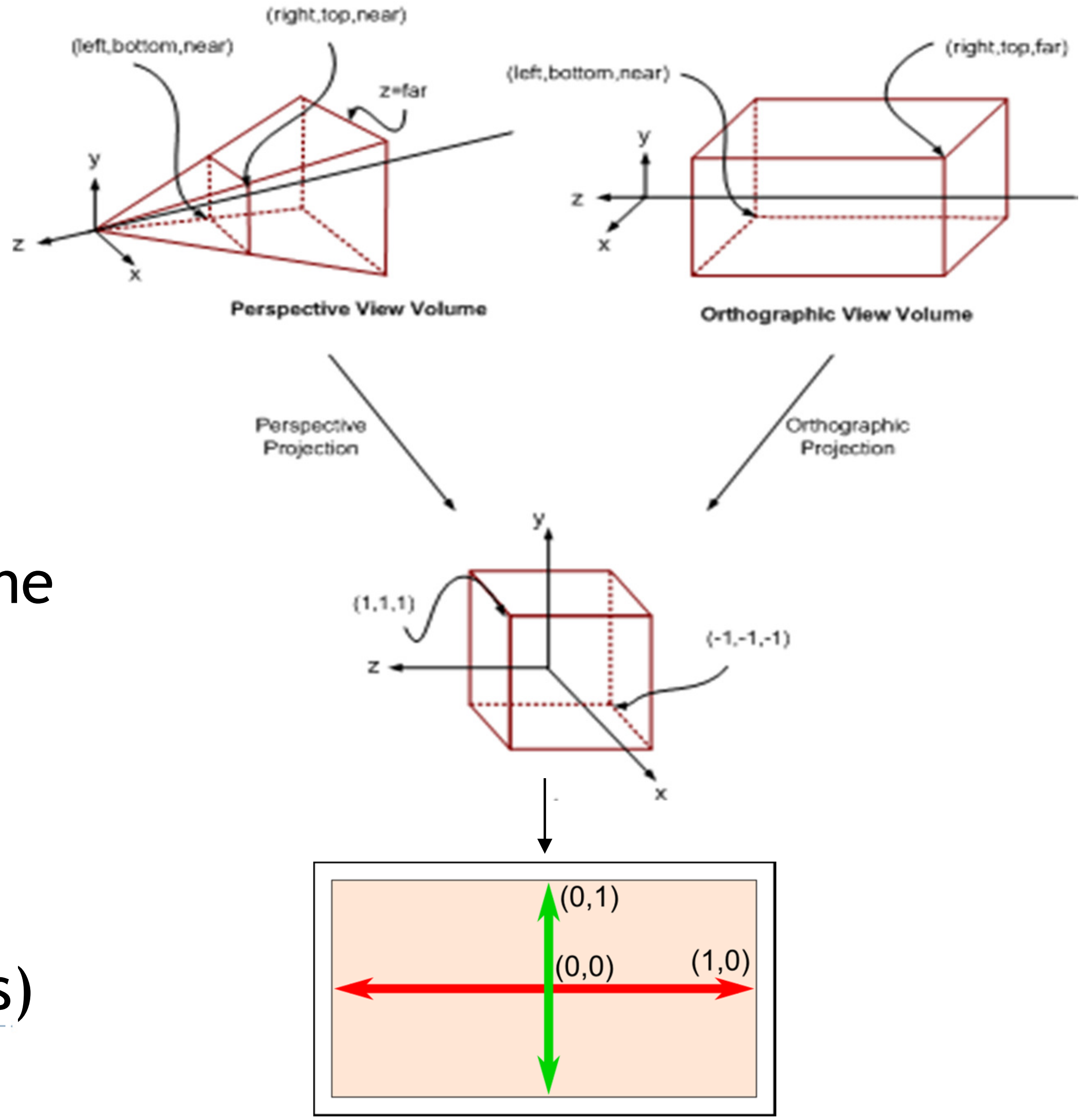
Camera coordinates

Projection matrix

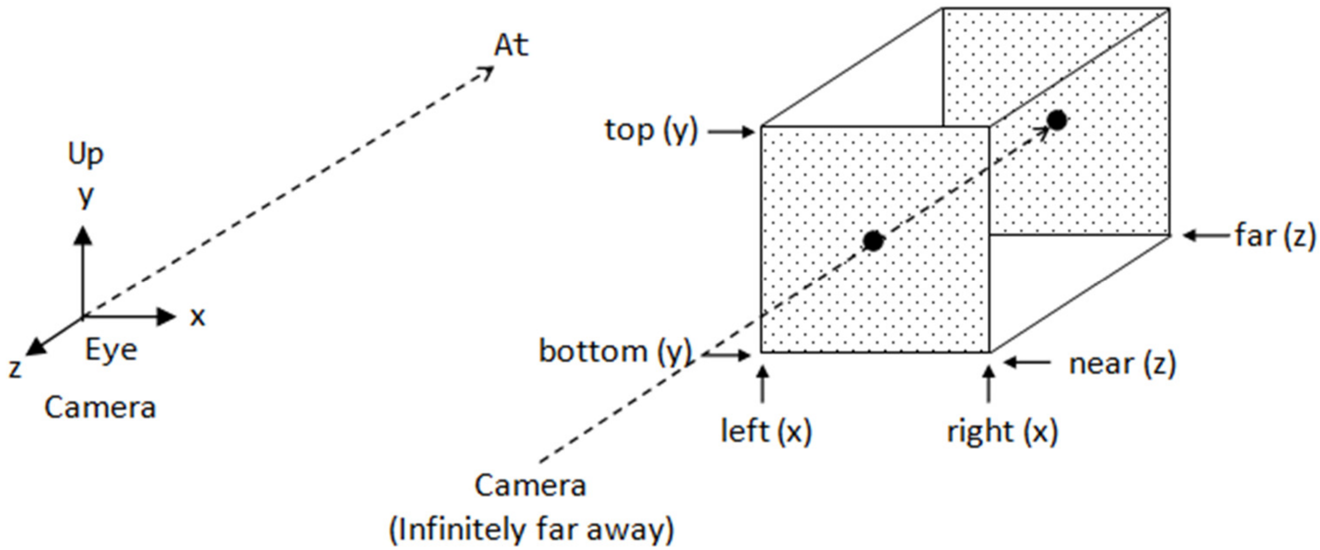
Canonical view volume

Viewport transformation

Image space
(pixel coordinates)

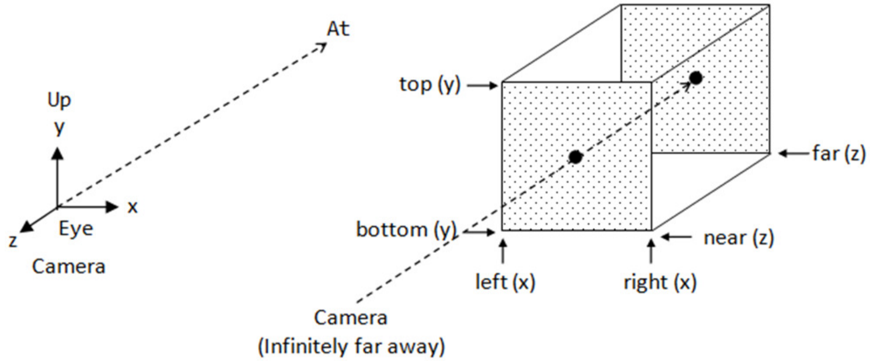


Orthographic View Volume



- Specified by 6 parameters:
  - Right, left, top, bottom, near, far
- Or, if symmetrical:
  - Width, height, near, far

Orthographic Projection Matrix



$$\mathbf{P}_{ortho}(right, left, top, bottom, near, far) = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In OpenGL:
glOrtho(left, right, bottom, top, near, far)

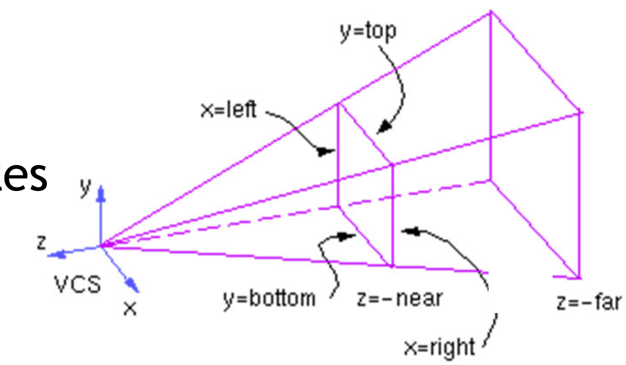
$$\mathbf{P}_{ortho}(width, height, near, far) = \begin{bmatrix} \frac{2}{width} & 0 & 0 & 0 \\ 0 & \frac{2}{height} & 0 & 0 \\ 0 & 0 & \frac{2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

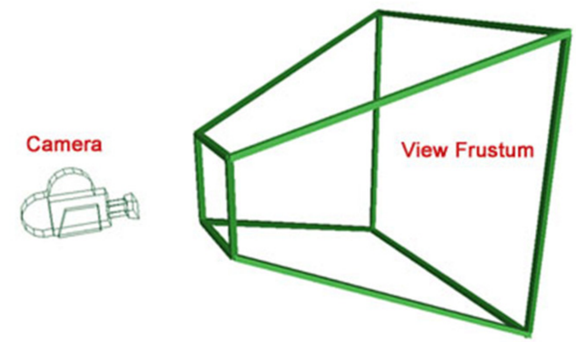
No equivalent in OpenGL

Perspective View Volume

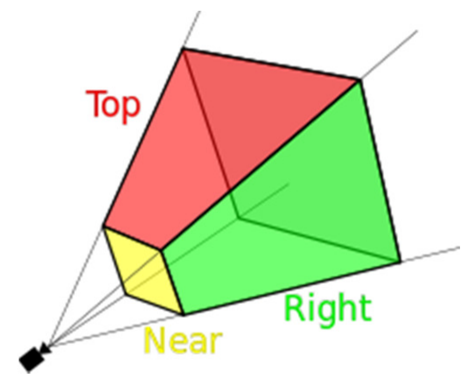
General view volume

Camera coordinates



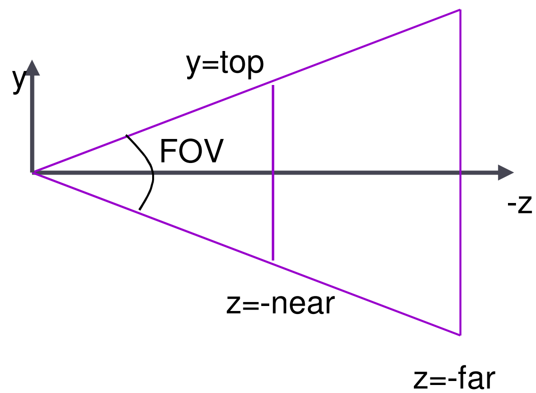


- Defined by 6 parameters, in camera coordinates
  - Left, right, top, bottom boundaries
  - Near, far clipping planes
- Clipping planes to avoid numerical problems
  - Divide by zero
  - Low precision for distant objects
- Usually symmetric, i.e., left=-right, top=-bottom



Perspective View Volume

Symmetrical view volume



- Only 4 parameters
  - Vertical field of view (FOV)
  - Image aspect ratio (width/height)
  - Near, far clipping planes

$$\text{aspect ratio} = \frac{right - left}{top - bottom} = \frac{right}{top}$$

$$\tan(FOV/2) = \frac{top}{near}$$

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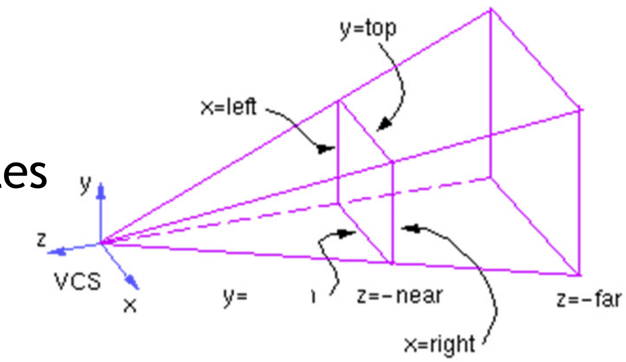
Perspective Projection Matrix

- General view frustum with 6 parameters

Camera coordinates



$$\mathbf{P}_{persp}(left, right, top, bottom, near, far) =$$

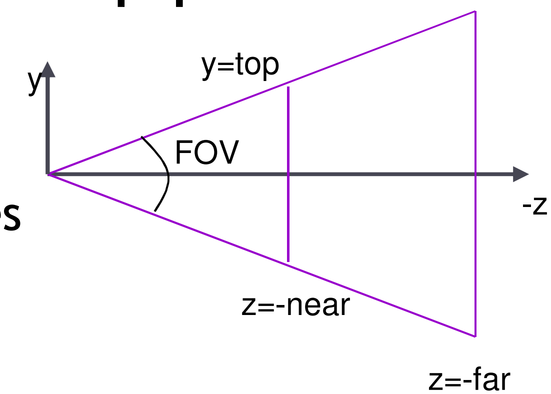
$$\begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & \frac{-(far+near)}{far-near} & \frac{-2far \cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

In OpenGL:
glFrustum(left, right, bottom, top, near, far)

Perspective Projection Matrix

- Symmetrical view frustum with field of view, aspect ratio, near and far clip planes

Camera coordinates



$$\mathbf{P}_{persp}(FOV, aspect, near, far) = \begin{bmatrix} \frac{1}{aspect \cdot \tan(FOV/2)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(FOV/2)} & 0 & 0 \\ 0 & 0 & \frac{near+far}{near-far} & \frac{2 \cdot near \cdot far}{near-far} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

In OpenGL:
gluPerspective(fov, aspect, near, far)

Canonical View Volume

- Goal: create projection matrix so that
  - User defined view volume is transformed into canonical view volume: cube [-1,1]x[-1,1]x[-1,1]
  - Multiplying corner vertices of view volume by projection matrix and performing homogeneous divide yields corners of canonical view volume
- Perspective and orthographic projection are treated the same way
- Canonical view volume is last stage in which coordinates are in 3D
  - Next step is projection to 2D frame buffer

Viewport Transformation

- After applying projection matrix, scene points are in *normalized viewing coordinates*
  - Per definition within range [-1..1] x [-1..1] x [-1..1]
- Next is projection from 3D to 2D (not reversible)
- Normalized viewing coordinates can be mapped to image (=pixel=frame buffer) coordinates
  - Range depends on window (view port) size: [x0…x1] x [y0…y1]
- Scale and translation required:

$$\mathbf{D}(x_0, x_1, y_0, y_1) = \begin{bmatrix} (x_1 - x_0)/2 & 0 & 0 & (x_0 + x_1)/2 \\ 0 & (y_1 - y_0)/2 & 0 & (y_0 + y_1)/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lecture Overview

- View Volumes
- Vertex Transformation
- Rendering Pipeline
- Culling

Complete Vertex Transformation

- Mapping a 3D point in object coordinates to pixel coordinates:

$$\mathbf{p}' = \mathbf{DPC}^{-1}\mathbf{M}\mathbf{p}$$

Object space

- **M**: Object-to-world matrix
- **C**: camera matrix
- **P**: projection matrix
- **D**: viewport matrix

Complete Vertex Transformation

- Mapping a 3D point in object coordinates to pixel coordinates:

$$\mathbf{p}' = \mathbf{D}\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$$

Object space

World space

  - **M**: Object-to-world matrix
  - **C**: camera matrix
  - **P**: projection matrix
  - **D**: viewport matrix

Complete Vertex Transformation

- Mapping a 3D point in object coordinates to pixel coordinates:

$$\mathbf{p}' = \mathbf{D}\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$$

Object space

World space

Camera space

  - **M**: Object-to-world matrix
  - **C**: camera matrix
  - **P**: projection matrix
  - **D**: viewport matrix

Complete Vertex Transformation

- Mapping a 3D point in object coordinates to pixel coordinates:

$$\mathbf{p}' = \mathbf{D}\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$$

- $\mathbf{p}$: Object space
- $\mathbf{M}\mathbf{p}$: World space
- $\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$: Camera space
- $\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$: Canonical view volume

  - **M**: Object-to-world matrix
  - **C**: camera matrix
  - **P**: projection matrix
  - **D**: viewport matrix

Complete Vertex Transformation

- Mapping a 3D point in object coordinates to pixel coordinates: $\mathbf{p}' = \mathbf{D}\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$

  Object space

  World space

  Camera space

  Canonical view volume

  Image space

  - **M**: Object-to-world matrix
  - **C**: camera matrix
  - **P**: projection matrix
  - **D**: viewport matrix

Complete Vertex Transformation

- Mapping a 3D point in object coordinates to pixel coordinates:

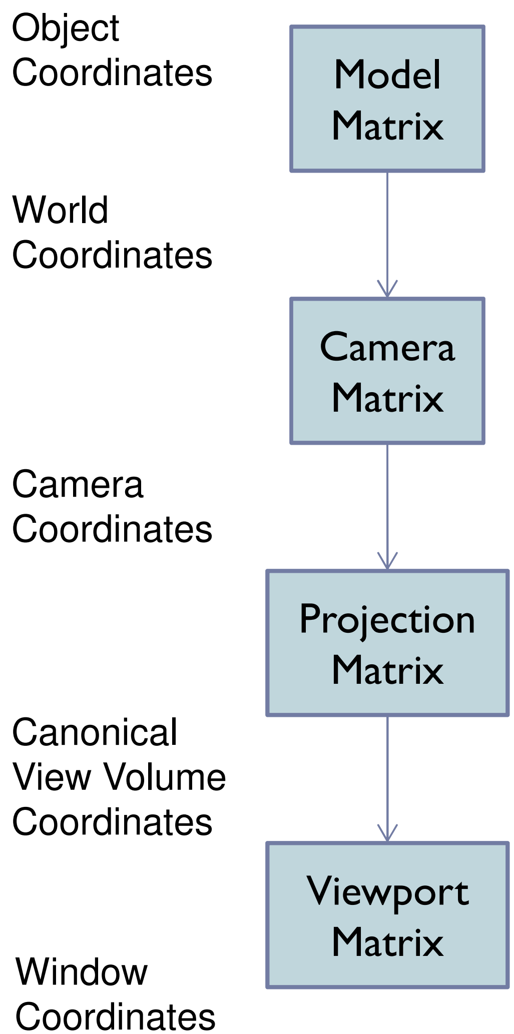
$$\mathbf{p}' = \mathbf{D}\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$$

$$\mathbf{p}' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix}$$

Pixel coordinates: x'/w', y'/w'

  - **M**: Object-to-world matrix
  - **C**: camera matrix
  - **P**: projection matrix
  - **D**: viewport matrix

The Complete Vertex Transformation



Complete Vertex Transformation in OpenGL

- Mapping a 3D point in object coordinates to pixel coordinates:

OpenGL GL_MODELVIEW matrix

$$\mathbf{p}' = \mathbf{D}\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$$

OpenGL GL_PROJECTION matrix

  - **M**: Object-to-world matrix
  - **C**: camera matrix
  - **P**: projection matrix
  - **D**: viewport matrix

Complete Vertex Transformation in OpenGL

- GL_MODELVIEW, $\mathbf{C^{-1}M}$
  - Defined by the programmer.
  - Think of the ModelView matrix as where you stand with the camera and the direction you point it.
- GL_PROJECTION, **P**
  - Utility routines to set it by specifying view volume: glFrustum(), gluPerspective(), glOrtho()
  - Think of the projection matrix as describing the attributes of your camera, such as field of view, focal length, etc.
- Viewport, **D**
  - Specify implicitly via glViewport()
  - No direct access with equivalent to GL_MODELVIEW or GL_PROJECTION