CSE 167:
Introduction to Computer Graphics
Lecture #3: Projection

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Announcements

- Project 1 due Friday September 30th, presentation in lab 260 starting 1:30pm
  - Both executable and source code required for grading. We will ask questions about the code!
  - List your name on the whiteboard once you get to the lab. Homework will be graded in this order.
- Project 2 is due Friday October 7th
  - Introduction by Jorge on Mon at 3pm in lab 260
- TA office hours on Thursdays: competition with cse132 and another class

- Remaining questions about Tuesday's lecture?

Lecture Overview

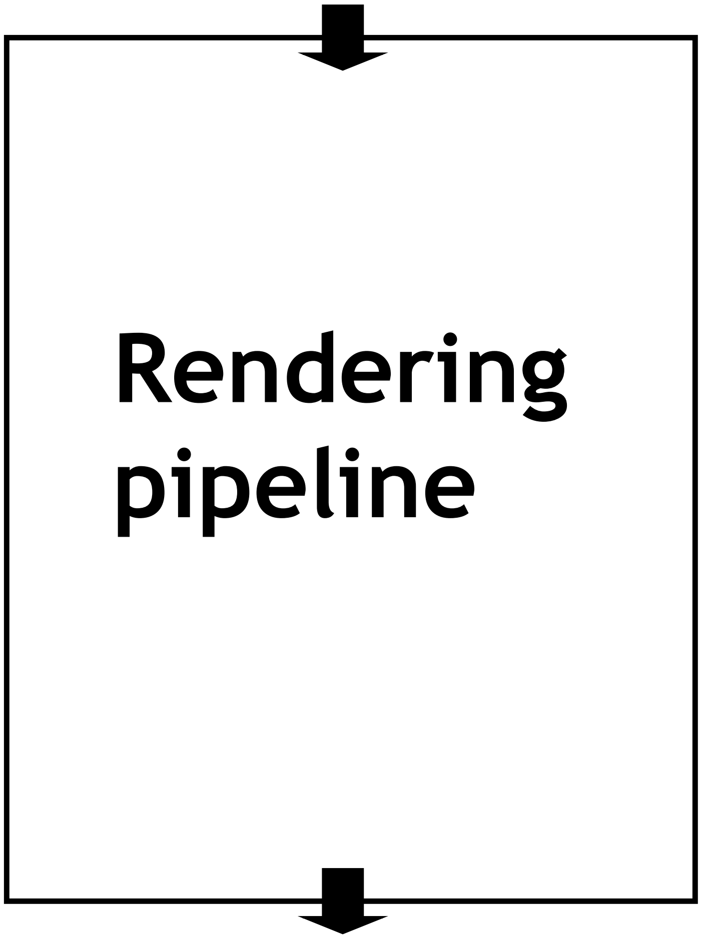
- Rendering Pipeline
- Projections
- View Volumes, Clipping

Objects in camera coordinates

- We have things lined up the way we like them on screen
  - x to the right
  - y up
  - $-z$ going into the screen
  - Objects to look at are in front of us, i.e. have negative z values
- But objects are still in 3D
- Problem: project them into 2D

Rendering Pipeline

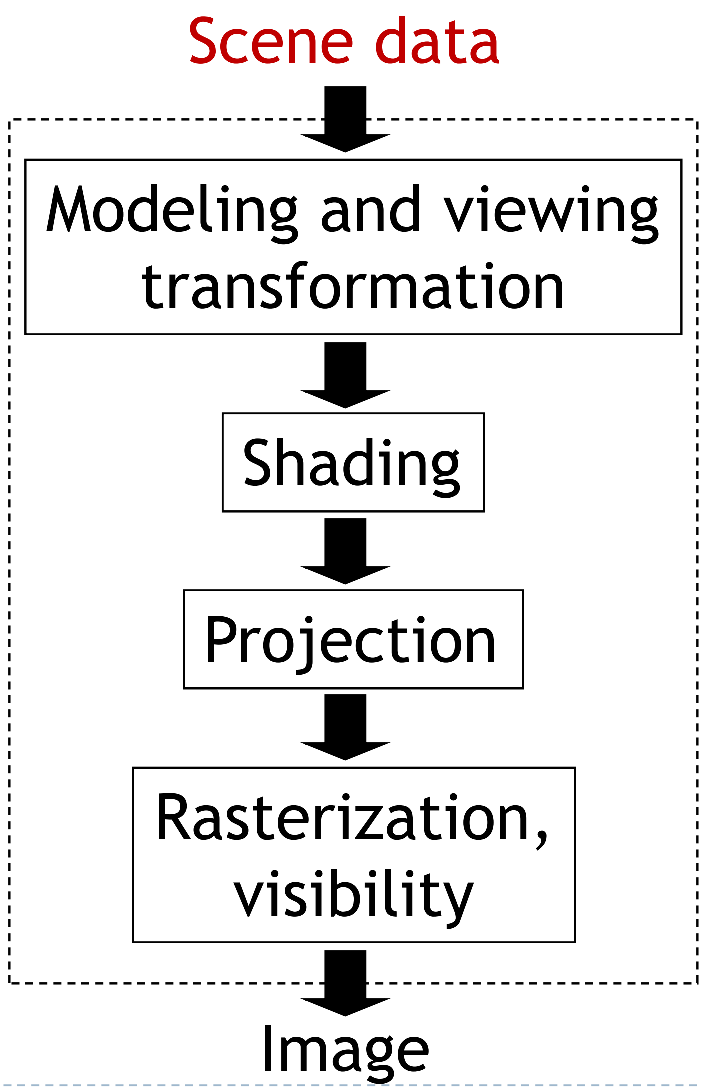
Scene data



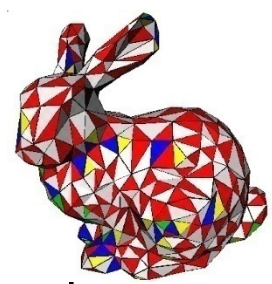
Image

- Hardware and software which draws 3D scenes on the screen
- Consists of several stages
  - Simplified version here
- Most operations performed by specialized hardware (GPU)
- Access to hardware through low-level 3D API (OpenGL, DirectX)
- All scene data flows through the pipeline at least once for each frame

Rendering Pipeline

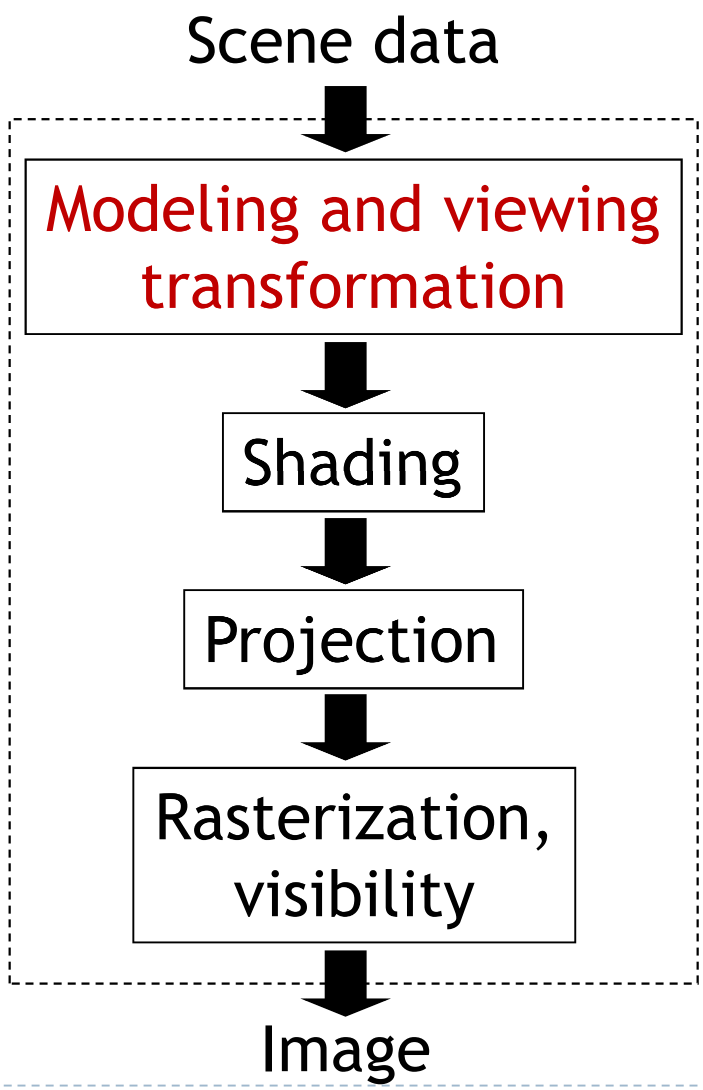


- Textures, lights, etc.
- Geometry
  - Vertices and how they are connected
  - Triangles, lines, points, triangle strips
  - Attributes such as color



- Specified in object coordinates
- Processed by the rendering pipeline one-by-one

Rendering Pipeline

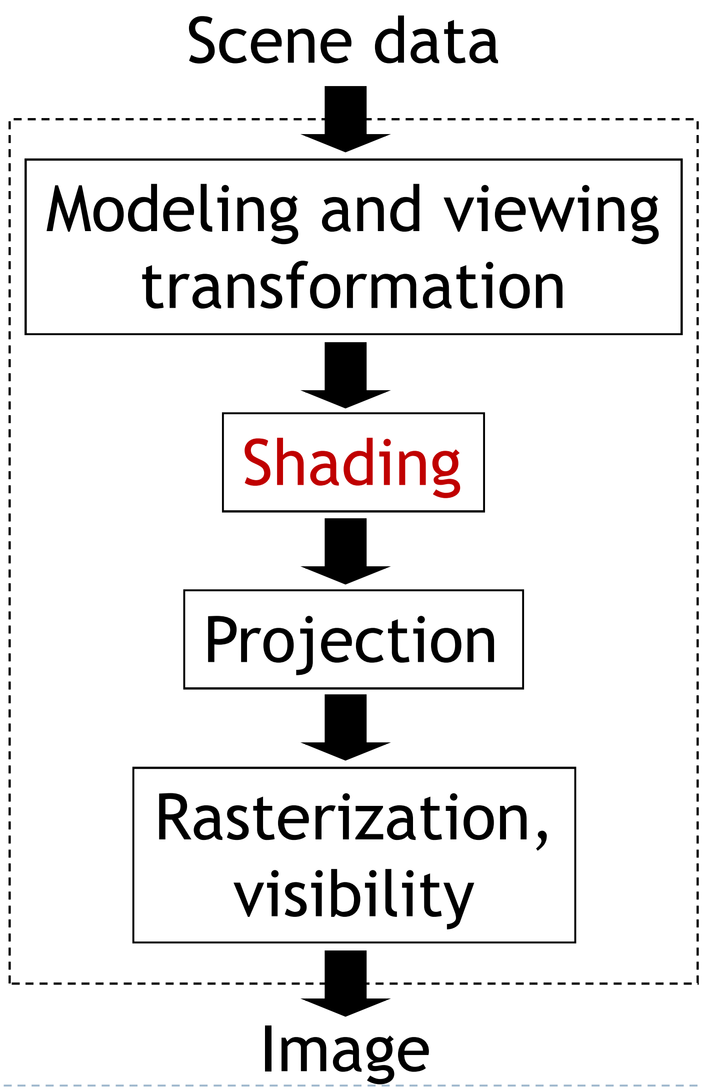


- Transform object to camera coordinates
- Specified by GL_MODELVIEW matrix in OpenGL
- User computes GL_MODELVIEW matrix as discussed

$$\mathbf{p}_{camera} = \mathbf{C}^{-1}\mathbf{M}\mathbf{p}_{object}$$

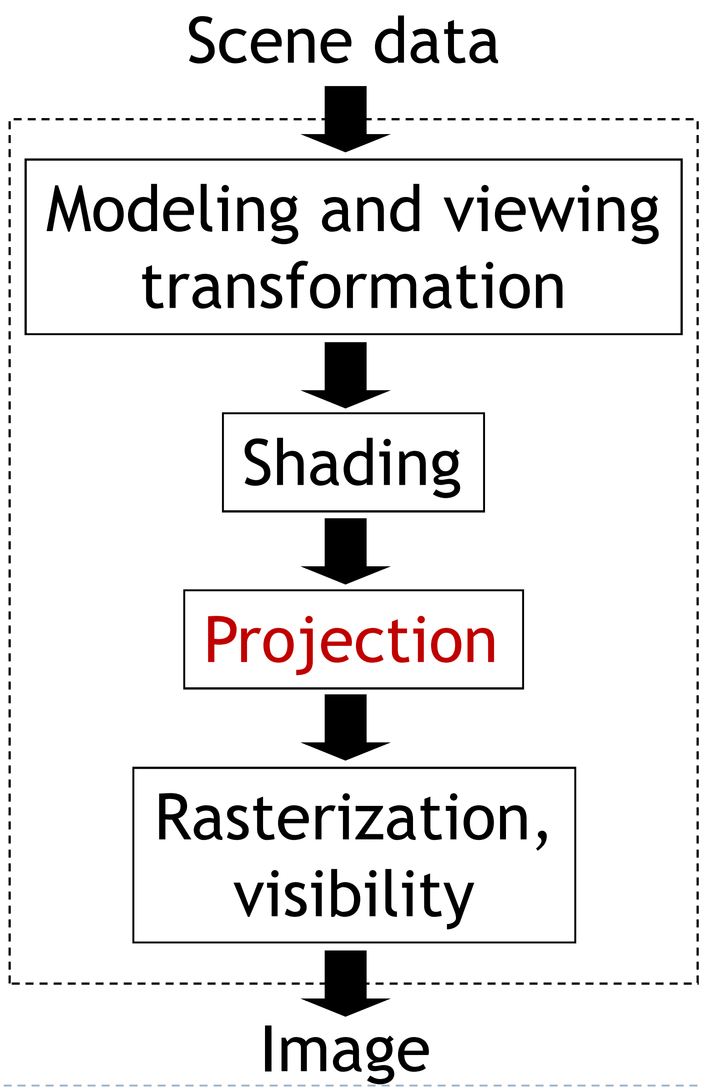
MODELVIEW matrix

Rendering Pipeline



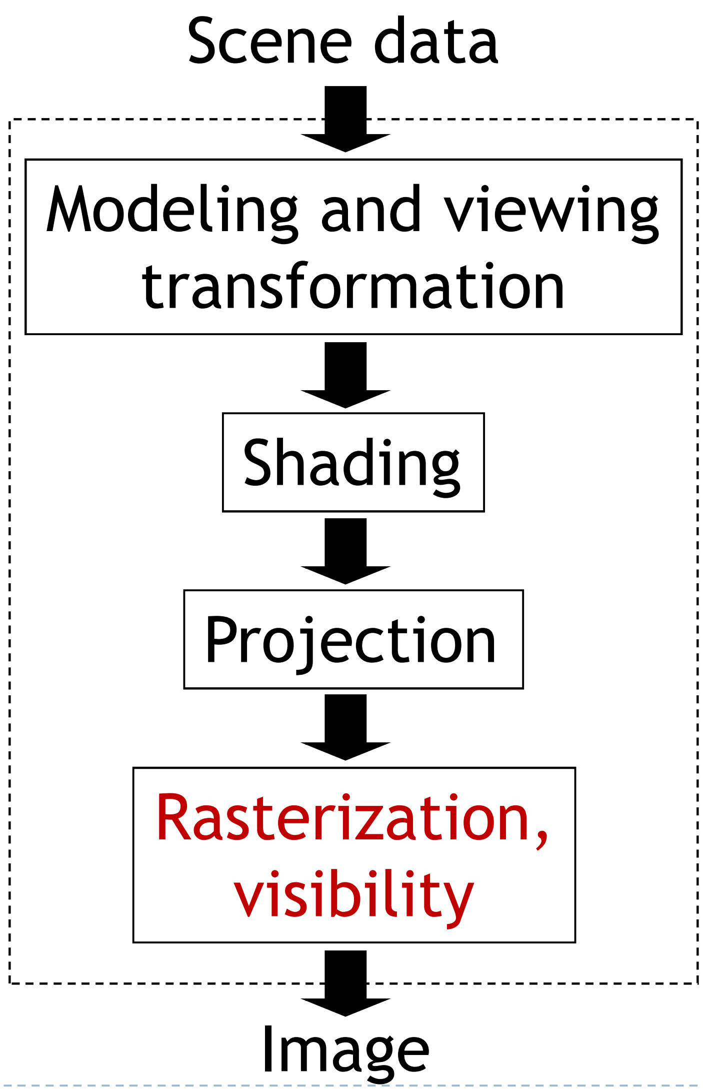
- Look up light sources
- Compute color for each vertex
- Covered later in the course

Rendering Pipeline

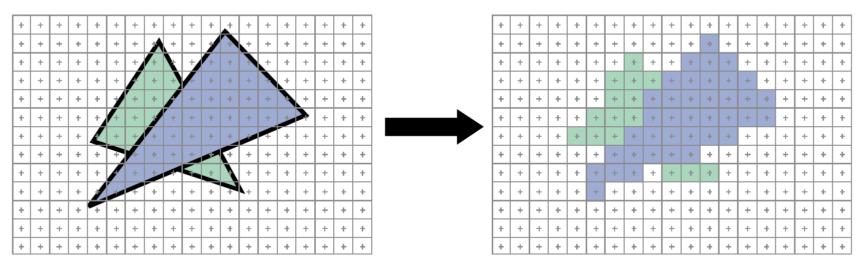


- Project 3D vertices to 2D image positions
- GL_PROJECTION matrix
- Covered in today's lecture

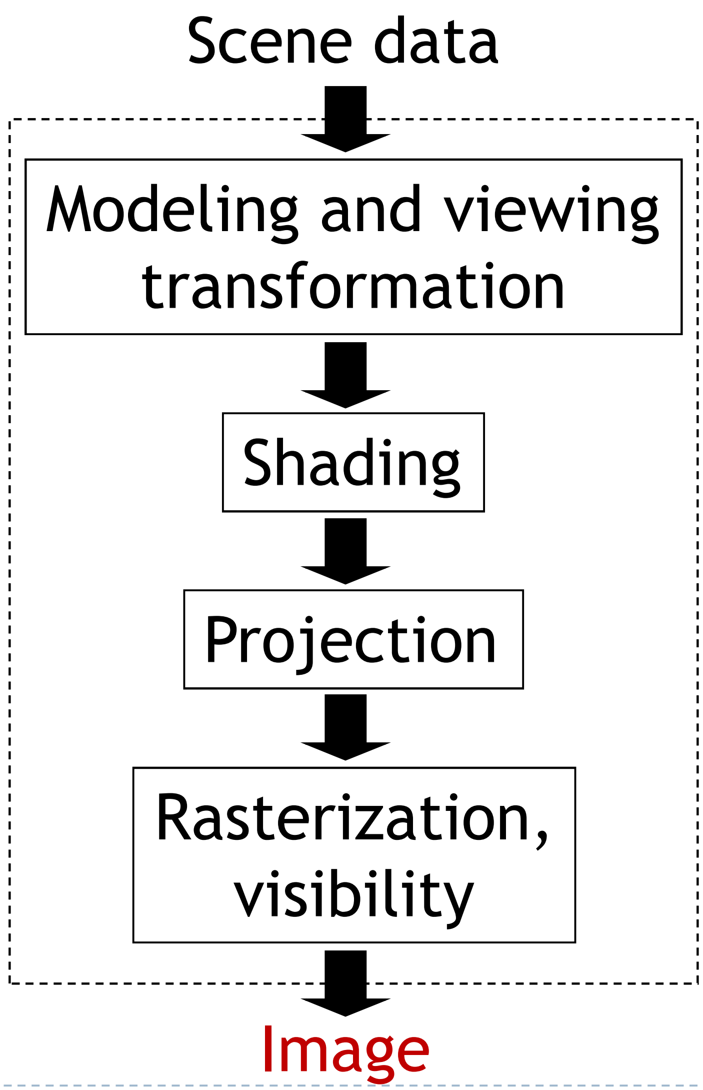
Rendering Pipeline



- Draw primitives (triangles, lines, etc.)
- Determine what is visible
- Covered in next lecture

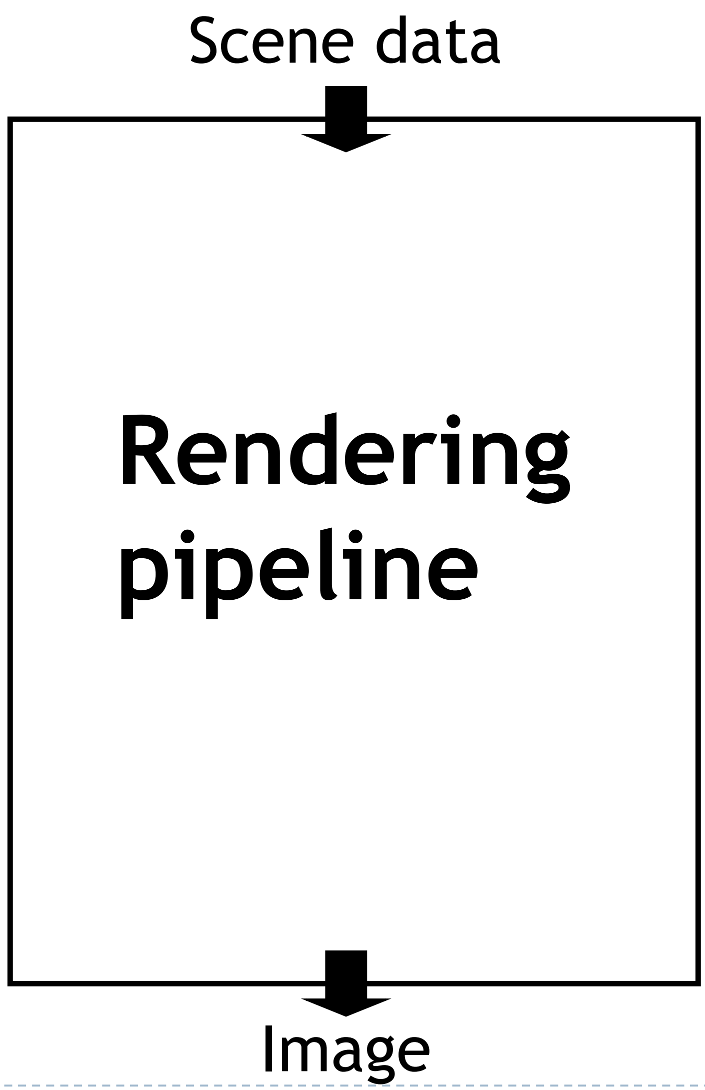


Rendering Pipeline



- Pixel colors

Rendering Engine



- Additional software layer encapsulating low-level API
- Higher level functionality than OpenGL
- Platform independent
- Layered software architecture common in industry
  - Game engines http://en.wikipedia.org/wiki/Game_engine

Lecture Overview

- Rendering Pipeline
- Projections
- View Volumes, Clipping

Projections

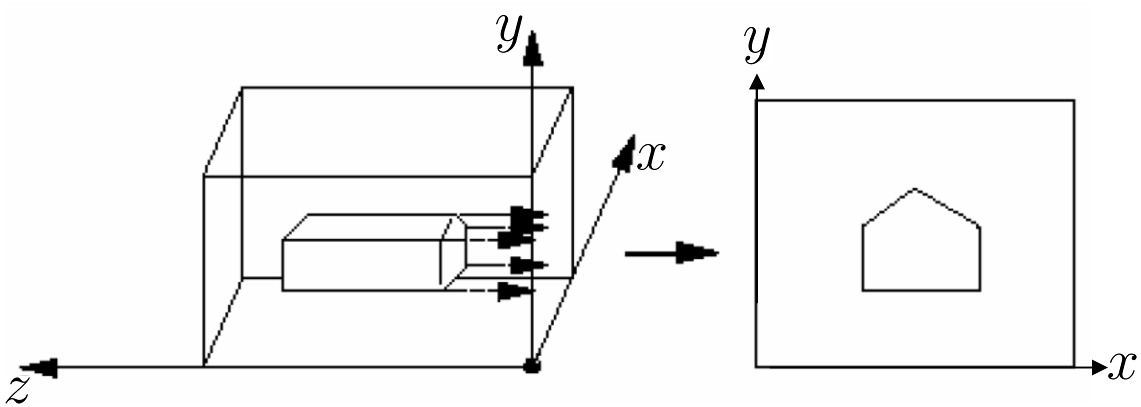
- Given 3D points (vertices) in camera coordinates, determine corresponding image coordinates

Orthographic Projection

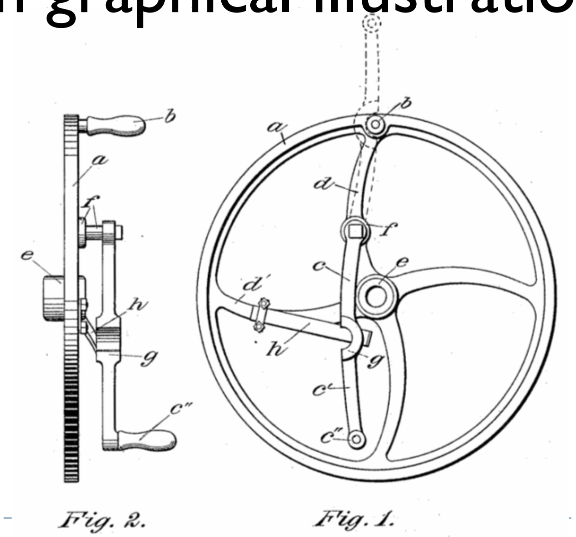
- a.k.a. Parallel Projection
- Done by ignoring z-coordinate
- Use camera space xy coordinates as image coordinates

Orthographic Projection

- Project points to x-y plane along parallel lines



- Used in graphical illustrations, architecture, 3D modeling

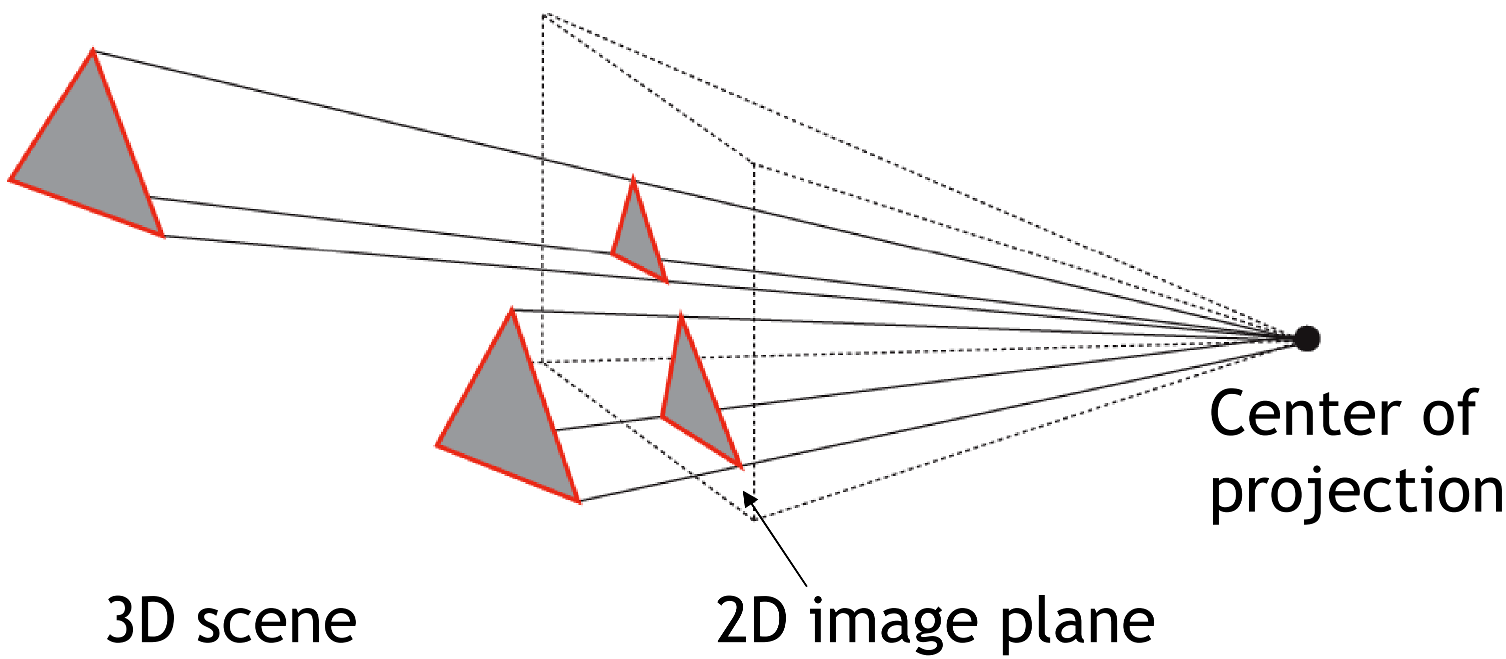


Perspective Projection

- Most common for computer graphics
- Simplified model of human eye, or camera lens (*pinhole camera*)
- Things farther away appear to be smaller
- Discovery attributed to Filippo Brunelleschi (Italian architect) in the early 1400's

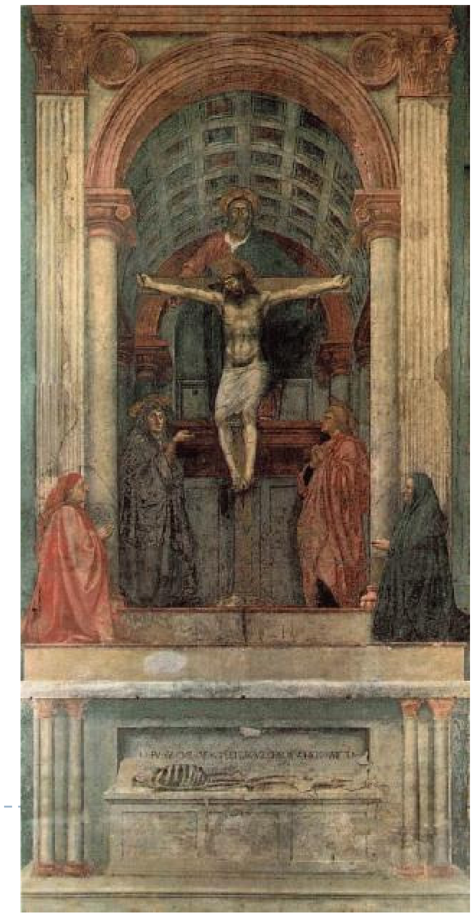
Perspective Projection

- Project along rays that converge in center of projection



Perspective Projection

Parallel lines are
no longer parallel,
converge in one point



Earliest example:
La Trinitá (1427) by Masaccio

Perspective Projection

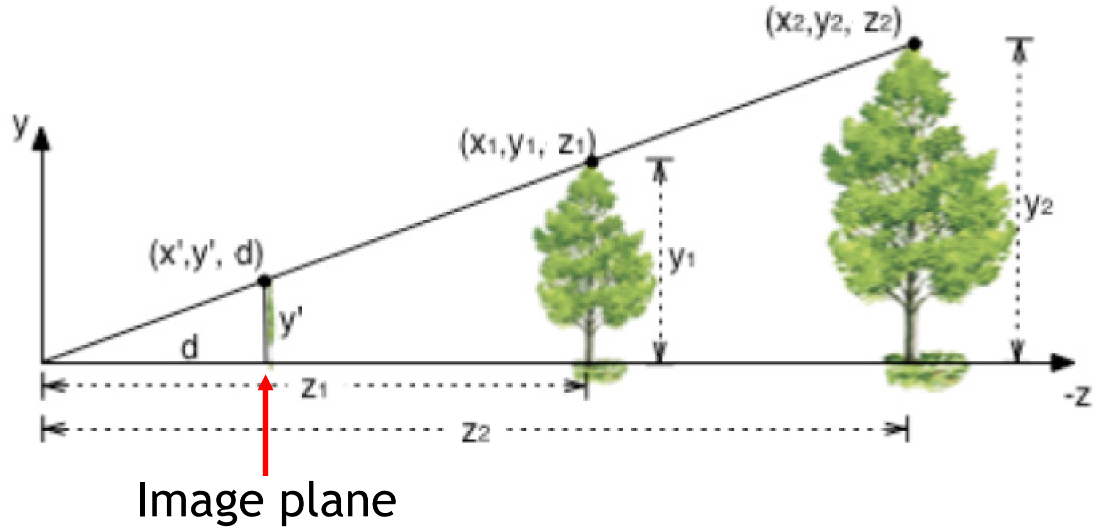
The math: simplified case

$$\frac{y'}{d} = \frac{y_1}{z_1}$$

$$y' = \frac{y_1 d}{z_1}$$

$$x' = \frac{x_1 d}{z_1}$$

$$z' = d$$



Perspective Projection

The math: simplified case

$$x' = \frac{x_1 d}{z_1}$$

$$y' = \frac{y_1 d}{z_1}$$

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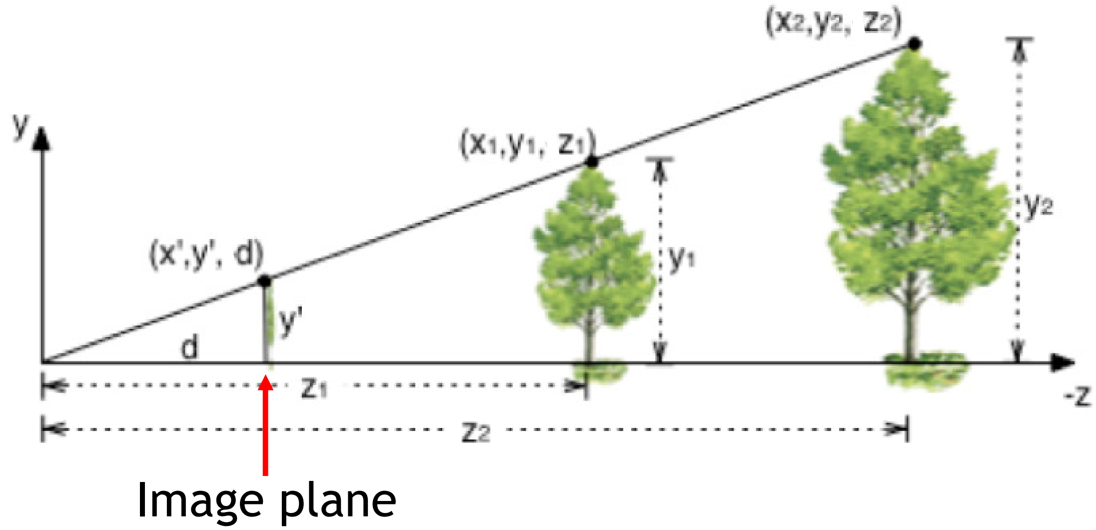


- We can express this using homogeneous coordinates and 4x4 matrices

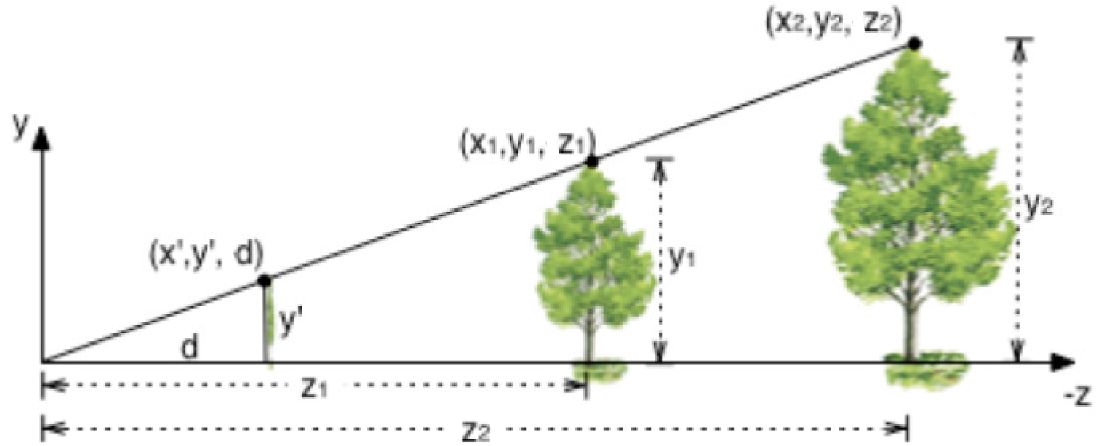
Perspective Projection

The math: simplified case

$$x' = \frac{x_1 d}{z_1}$$

$$y' = \frac{y_1 d}{z_1}$$

$$z' = d$$



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \Rightarrow \begin{bmatrix} xd/z \\ yd/z \\ d \\ 1 \end{bmatrix}$$

Projection matrix Homogeneous division

Perspective Projection

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} xd/z \\ yd/z \\ d \\ 1 \end{bmatrix}$$

Projection matrix Homogeneous division

- Using projection matrix, homogeneous division seems more complicated than just multiplying all coordinates by d/z, so why do it?
- It will allow us to:
  - handle different types of projections in a unified way
  - define arbitrary view volumes
- Divide by w (perspective division, homogeneous division) after performing projection transform
  - Graphics hardware does this automatically

Photorealistic Rendering

- More than just perspective projection
- Some effects are too complex for hardware rendering
- For example: lens effects

Focus, depth of field

Fish-eye lens

Photorealistic Rendering

Chromatic Aberration

Motion Blur

Lecture Overview

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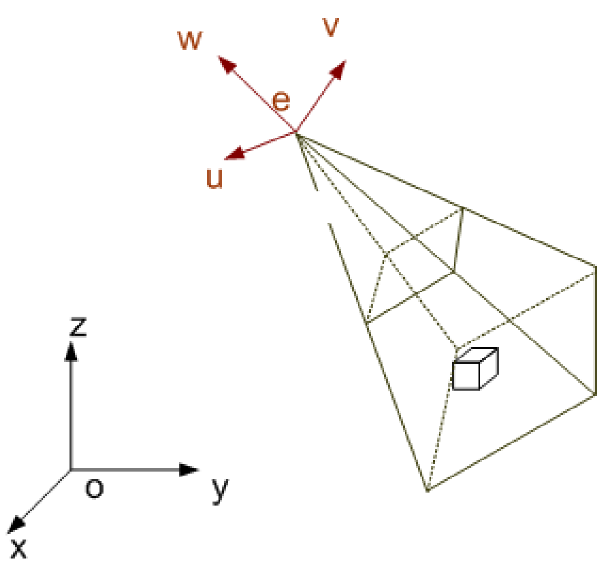
View Volumes

- Define 3D volume seen by camera

Perspective view volume

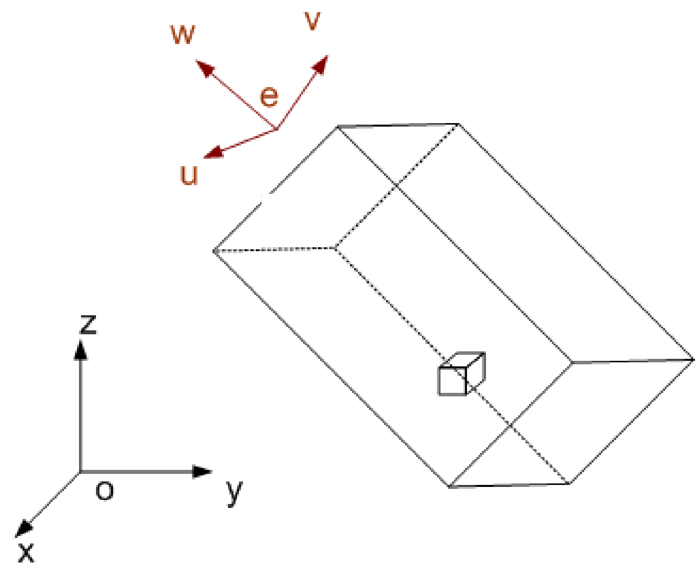
Camera coordinates



World coordinates

Orthographic view volume

Camera coordinates

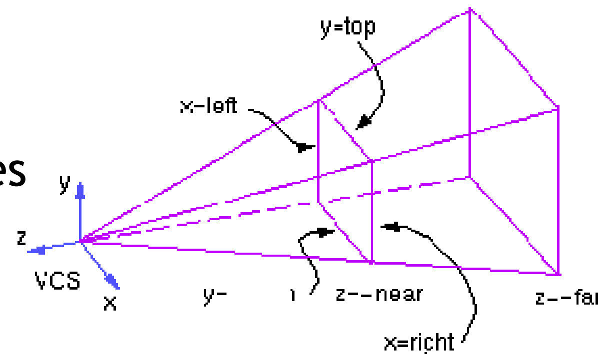


World coordinates

Perspective View Volume

General view volume

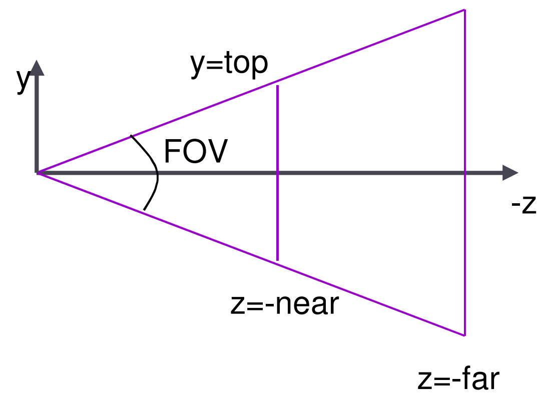
Camera
coordinates



- Defined by 6 parameters, in camera coordinates
  - Left, right, top, bottom boundaries
  - Near, far clipping planes
- Clipping planes to avoid numerical problems
  - Divide by zero
  - Low precision for distant objects
- Usually symmetric, i.e., left=-right, top=-bottom

Perspective View Volume

Symmetrical view volume

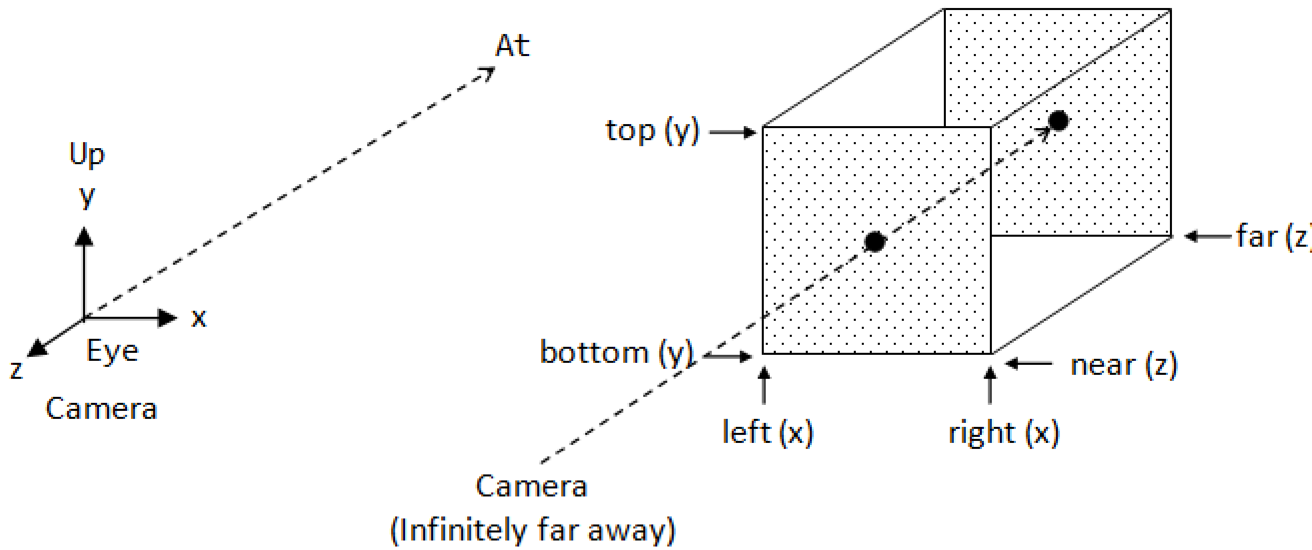


- Only 4 parameters
  - Vertical field of view (FOV)
  - Image aspect ratio (width/height)
  - Near, far clipping planes

$$\text{aspect ratio}=\frac{right-left}{top-bottom}=\frac{right}{top}$$

$$\tan(FOV/2)=\frac{top}{near}$$

Orthographic View Volume



- Parameterized by 6 parameters
  - Right, left, top, bottom, near, far
- Or if symmetrical:
  - Width, height, near, far

Clipping

- Need to identify objects outside view volume
  - Avoid division by zero
  - Efficiency: don't draw objects outside view volume (view frustum culling)
- Performed in hardware
- Hardware always clips to the *canonical view volume:* cube [-1..1]x[-1..1]x[-1..1] centered at origin
- Need to transform **desired** view frustum to **canonical** view frustum