

CSE 167:  
Introduction to Computer Graphics  
Lecture #12: GLSL

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# Announcements

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- ▶ Project 5 due Friday at 3:30pm
- ▶ 2 REU positions for high speed networking
  - ▶ Under Dr. Thomas DeFanti

# Overview

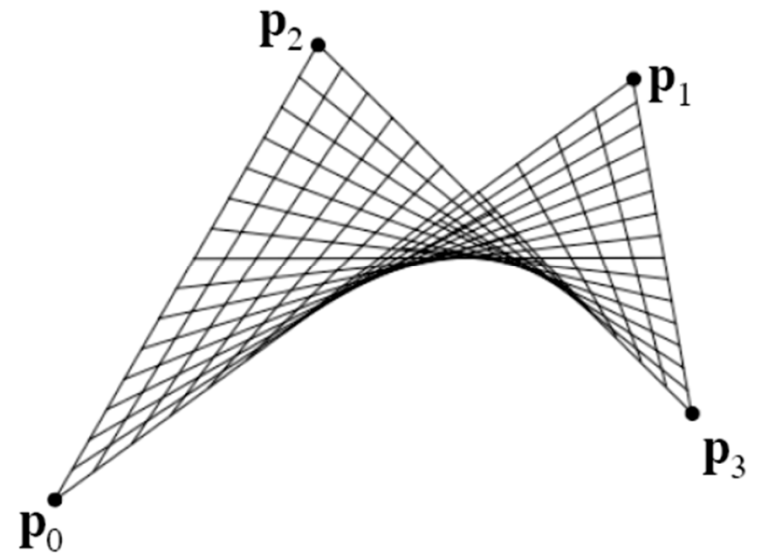
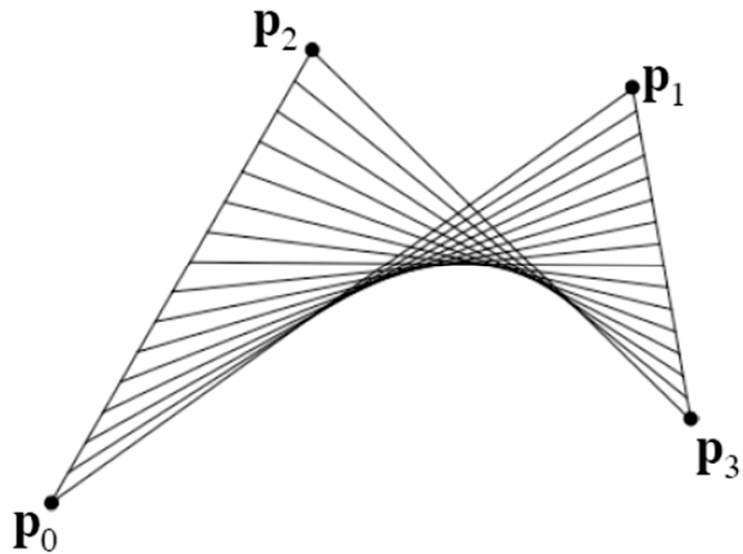
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- ▶ **Bi-linear patch**
- ▶ Bi-cubic Bézier patch
- ▶ Advanced parametric surfaces

# Bilinear Patch

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## ► Visualization



# Bilinear Patches

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- ▶ **Weighted sum of control points**

$$\mathbf{x}(u, v) = (1-u)(1-v)\mathbf{p}_0 + u(1-v)\mathbf{p}_1 + (1-u)v\mathbf{p}_2 + uv\mathbf{p}_3$$

- ▶ **Bilinear polynomial**

$$\mathbf{x}(u, v) = (\mathbf{p}_0 - \mathbf{p}_1 - \mathbf{p}_2 + \mathbf{p}_3)uv + (\mathbf{p}_1 - \mathbf{p}_0)u + (\mathbf{p}_2 - \mathbf{p}_0)v + \mathbf{p}_0$$

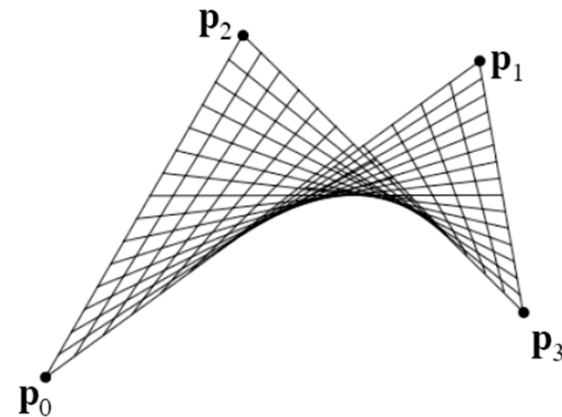
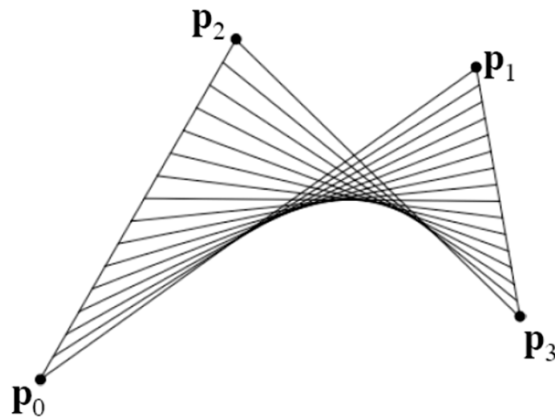
- ▶ **Matrix form**

$$\mathbf{x}(u, v) = \begin{bmatrix} 1-u & u \end{bmatrix} \begin{bmatrix} p_0 & p_2 \\ p_1 & p_3 \end{bmatrix} \begin{bmatrix} 1-v \\ v \end{bmatrix}$$

# Properties

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- ▶ Interpolates the control points
- ▶ The boundaries are straight line segments
- ▶ If all 4 points of the control mesh are co-planar, the patch is flat
- ▶ If the points are not co-planar, we get a curved surface
  - ▶ saddle shape (hyperbolic paraboloid)
- ▶ *The parametric curves are all straight line segments!*
  - ▶ a (doubly) ruled surface: has (two) straight lines through every point



- ▶ Not terribly useful as a modeling primitive

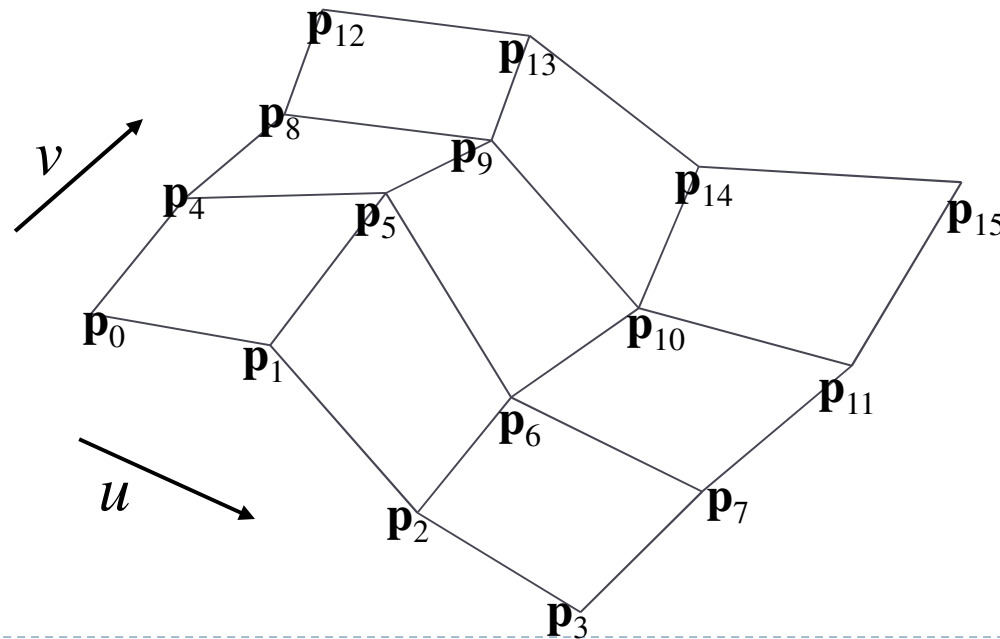
# Overview

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- ▶ Bi-linear patch
- ▶ Bi-cubic Bézier patch
- ▶ Advanced parametric surfaces

# Bicubic Bézier patch

- ▶ Grid of 4x4 control points,  $\mathbf{p}_0$  through  $\mathbf{p}_{15}$
- ▶ Four rows of control points define Bézier curves along  $u$   
 $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ ;  $\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7$ ;  $\mathbf{p}_8, \mathbf{p}_9, \mathbf{p}_{10}, \mathbf{p}_{11}$ ;  $\mathbf{p}_{12}, \mathbf{p}_{13}, \mathbf{p}_{14}, \mathbf{p}_{15}$
- ▶ Four columns define Bézier curves along  $v$   
 $\mathbf{p}_0, \mathbf{p}_4, \mathbf{p}_8, \mathbf{p}_{12}$ ;  $\mathbf{p}_1, \mathbf{p}_5, \mathbf{p}_9, \mathbf{p}_{13}$ ;  $\mathbf{p}_2, \mathbf{p}_6, \mathbf{p}_{10}, \mathbf{p}_{14}$ ;  $\mathbf{p}_3, \mathbf{p}_7, \mathbf{p}_{11}, \mathbf{p}_{15}$

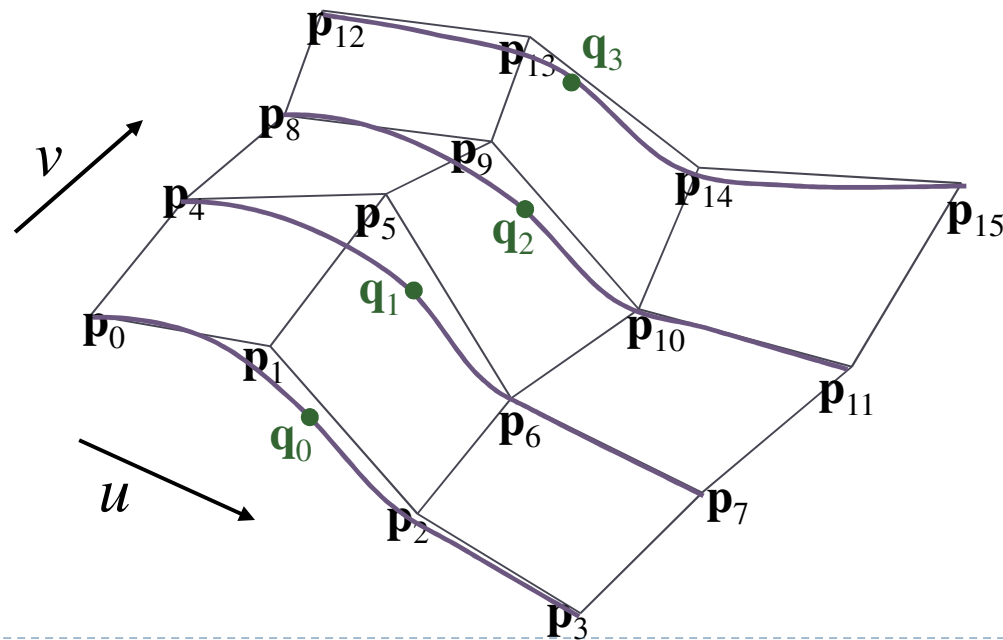




# Bézier Patch (Step 1)

- ▶ Evaluate four  $u$ -direction Bézier curves at scalar value  $u$   $[0..1]$

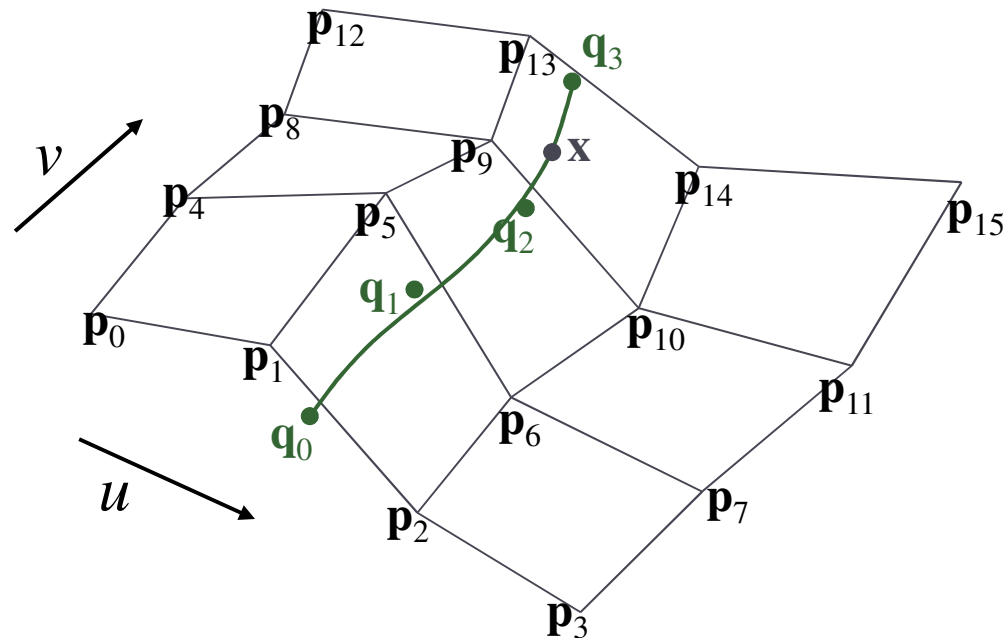
- ▶ Get points  $\mathbf{q}_0 \dots \mathbf{q}_3$   
 $\mathbf{q}_0 = \text{Bez}(u, \mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$   
 $\mathbf{q}_1 = \text{Bez}(u, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7)$   
 $\mathbf{q}_2 = \text{Bez}(u, \mathbf{p}_8, \mathbf{p}_9, \mathbf{p}_{10}, \mathbf{p}_{11})$   
 $\mathbf{q}_3 = \text{Bez}(u, \mathbf{p}_{12}, \mathbf{p}_{13}, \mathbf{p}_{14}, \mathbf{p}_{15})$



## Bézier Patch (Step 2)

- ▶ Points  $\mathbf{q}_0 \dots \mathbf{q}_3$  define a Bézier curve
- ▶ Evaluate it at  $v \in [0..1]$

$$\mathbf{x}(u, v) = \text{Bez}(v, \mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$$



# Bézier Patch

- Same result in either order (evaluate  $u$  before  $v$  or vice versa)

$$\mathbf{q}_0 = \text{Bez}(u, \mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$$

$$\mathbf{q}_1 = \text{Bez}(u, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7)$$

$$\mathbf{q}_2 = \text{Bez}(u, \mathbf{p}_8, \mathbf{p}_9, \mathbf{p}_{10}, \mathbf{p}_{11}) \Leftrightarrow$$

$$\mathbf{q}_3 = \text{Bez}(u, \mathbf{p}_{12}, \mathbf{p}_{13}, \mathbf{p}_{14}, \mathbf{p}_{15})$$

$$\mathbf{r}_0 = \text{Bez}(v, \mathbf{p}_0, \mathbf{p}_4, \mathbf{p}_8, \mathbf{p}_{12})$$

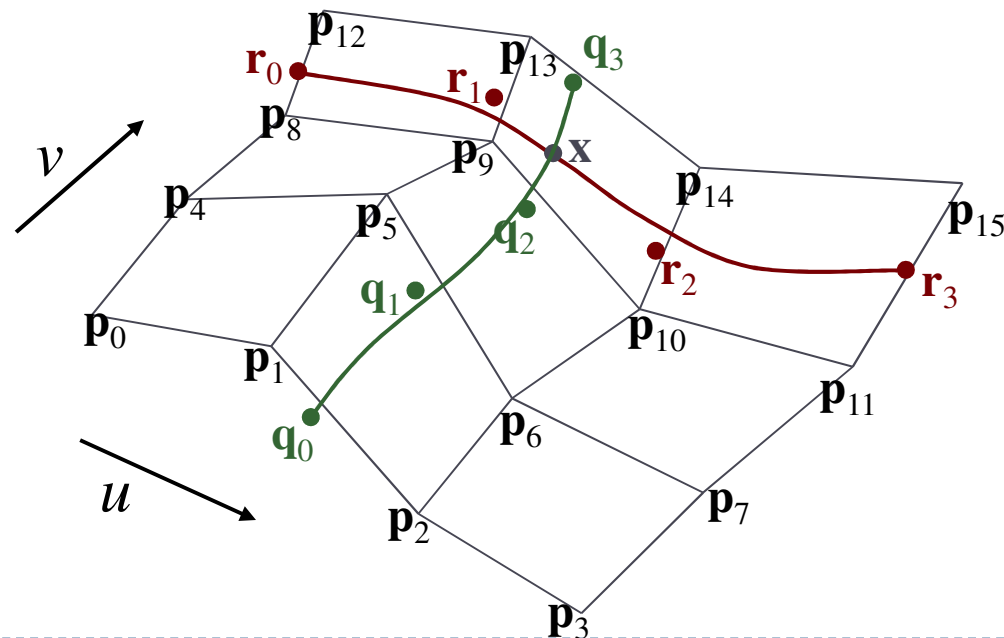
$$\mathbf{r}_1 = \text{Bez}(v, \mathbf{p}_1, \mathbf{p}_5, \mathbf{p}_9, \mathbf{p}_{13})$$

$$\mathbf{r}_2 = \text{Bez}(v, \mathbf{p}_2, \mathbf{p}_6, \mathbf{p}_{10}, \mathbf{p}_{14})$$

$$\mathbf{r}_3 = \text{Bez}(v, \mathbf{p}_3, \mathbf{p}_7, \mathbf{p}_{11}, \mathbf{p}_{15})$$

$$\mathbf{x}(u, v) = \text{Bez}(v, \mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$$

$$\mathbf{x}(u, v) = \text{Bez}(u, \mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$



# Bézier Patch: Matrix Form

$$\mathbf{U} = \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} v^3 \\ v^2 \\ v \\ 1 \end{bmatrix}$$

$$\mathbf{B}_{Bez} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \mathbf{B}_{Bez}^T$$

$$\mathbf{C}_x = \mathbf{B}_{Bez}^T \mathbf{G}_x \mathbf{B}_{Bez}$$

$$\mathbf{C}_y = \mathbf{B}_{Bez}^T \mathbf{G}_y \mathbf{B}_{Bez}$$

$$\mathbf{C}_z = \mathbf{B}_{Bez}^T \mathbf{G}_z \mathbf{B}_{Bez}$$

$$\mathbf{G}_x = \begin{bmatrix} p_{0x} & p_{1x} & p_{2x} & p_{3x} \\ p_{4x} & p_{5x} & p_{6x} & p_{7x} \\ p_{8x} & p_{9x} & p_{10x} & p_{11x} \\ p_{12x} & p_{13x} & p_{14x} & p_{15x} \end{bmatrix}, \quad \mathbf{G}_y = \dots, \quad \mathbf{G}_z = \dots$$

$$\mathbf{x}(u, v) = \begin{bmatrix} \mathbf{V}^T \mathbf{C}_x \mathbf{U} \\ \mathbf{V}^T \mathbf{C}_y \mathbf{U} \\ \mathbf{V}^T \mathbf{C}_z \mathbf{U} \end{bmatrix}$$

# Bézier Patch: Matrix Form

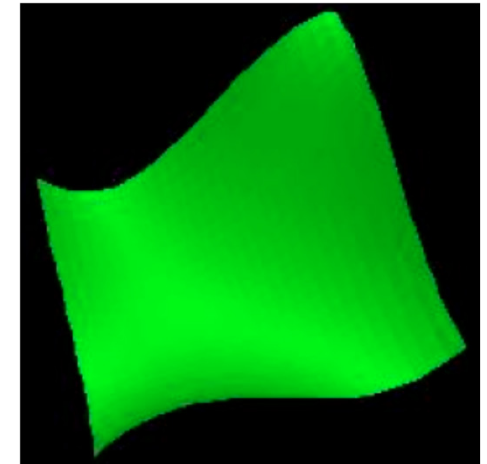
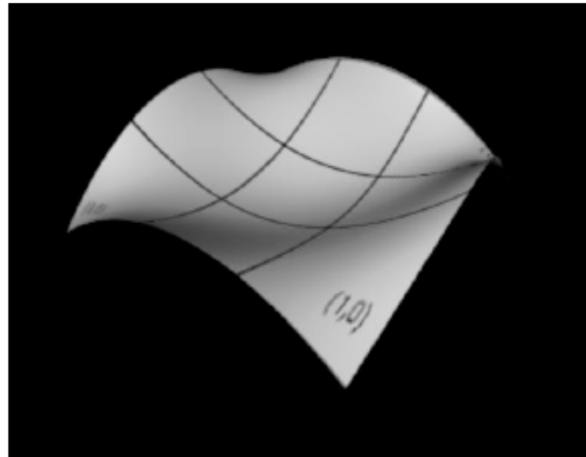
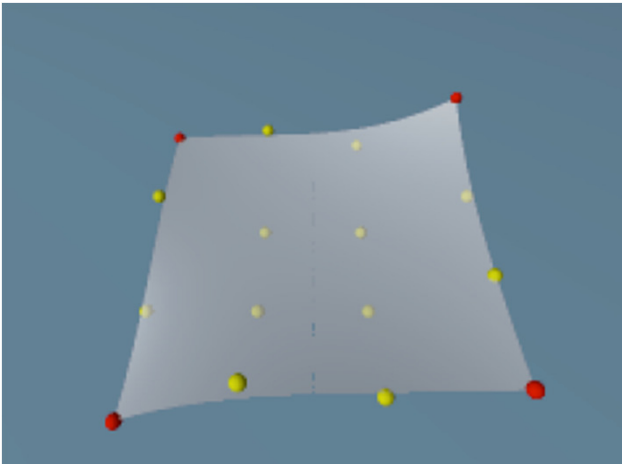
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- ▶  $\mathbf{C}_x$  stores the coefficients of the bicubic equation for  $x$
  - ▶  $\mathbf{C}_y$  stores the coefficients of the bicubic equation for  $y$
  - ▶  $\mathbf{C}_z$  stores the coefficients of the bicubic equation for  $z$
  - ▶  $\mathbf{G}_x$  stores the geometry ( $x$  components of the control points)
  - ▶  $\mathbf{G}_y$  stores the geometry ( $y$  components of the control points)
  - ▶  $\mathbf{G}_z$  stores the geometry ( $z$  components of the control points)
  - ▶  $\mathbf{B}_{\text{Bez}}$  is the basis matrix (Bézier basis)
  - ▶  $\mathbf{U}$  and  $\mathbf{V}$  are the vectors formed from the powers of  $u$  and  $v$
- 
- ▶ Compact notation
  - ▶ Leads to efficient method of computation
  - ▶ Can take advantage of hardware support for 4x4 matrix arithmetic

# Properties

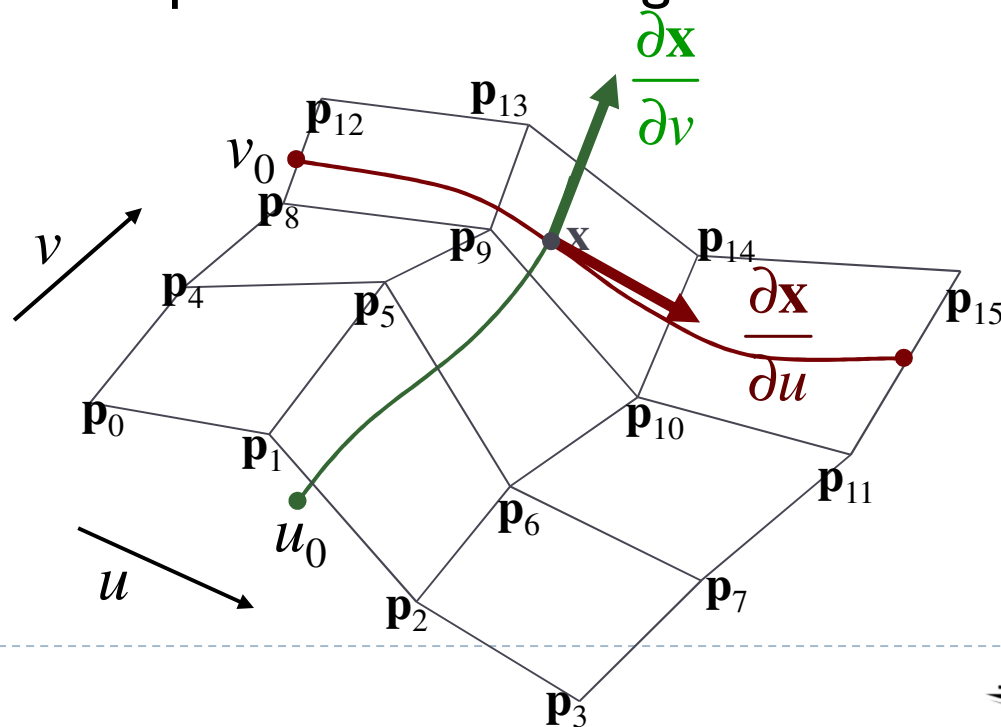
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- ▶ Convex hull: any point on the surface will fall within the convex hull of the control points
- ▶ Interpolates 4 corner points
- ▶ Approximates other 12 points, which act as “handles”
- ▶ The boundaries of the patch are the Bézier curves defined by the points on the mesh edges
- ▶ The parametric curves are all Bézier curves



# Tangents of a Bézier patch

- ▶ Remember parametric curves  $\mathbf{x}(u, v_0)$ ,  $\mathbf{x}(u_0, v)$  where  $v_0, u_0$  is fixed
- ▶ Tangents to surface = tangents to parametric curves
- ▶ Tangents are partial derivatives of  $\mathbf{x}(u, v)$
- ▶ Normal is cross product of the tangents



# Tangents of a Bézier patch

$$\mathbf{q}_0 = \text{Bez}(u, \mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$$

$$\mathbf{q}_1 = \text{Bez}(u, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7)$$

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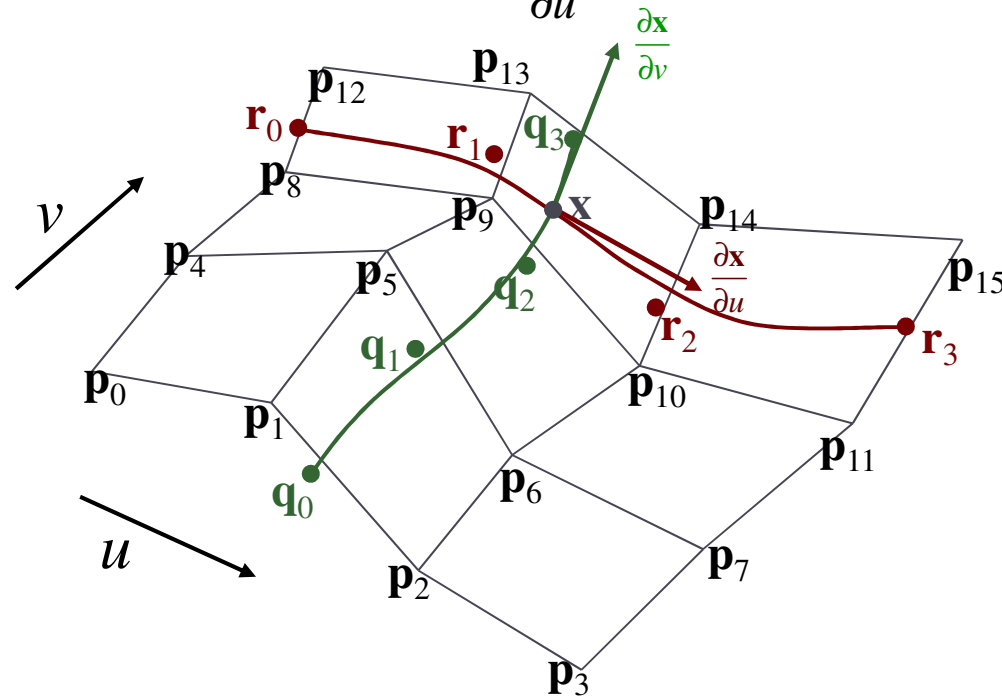
$$\mathbf{r}_1 = \text{Bez}(v, \mathbf{p}_1, \mathbf{p}_5, \mathbf{p}_9, \mathbf{p}_{13})$$

$$\mathbf{r}_2 = \text{Bez}(v, \mathbf{p}_2, \mathbf{p}_6, \mathbf{p}_{10}, \mathbf{p}_{14})$$

$$\mathbf{r}_3 = \text{Bez}(v, \mathbf{p}_3, \mathbf{p}_7, \mathbf{p}_{11}, \mathbf{p}_{15})$$

$$\frac{\partial \mathbf{x}}{\partial v}(u, v) = \text{Bez}'(v, \mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$$

$$\frac{\partial \mathbf{x}}{\partial u}(u, v) = \text{Bez}'(u, \mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

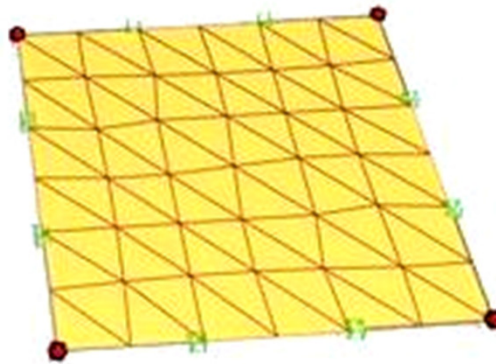




# Tessellating a Bézier patch

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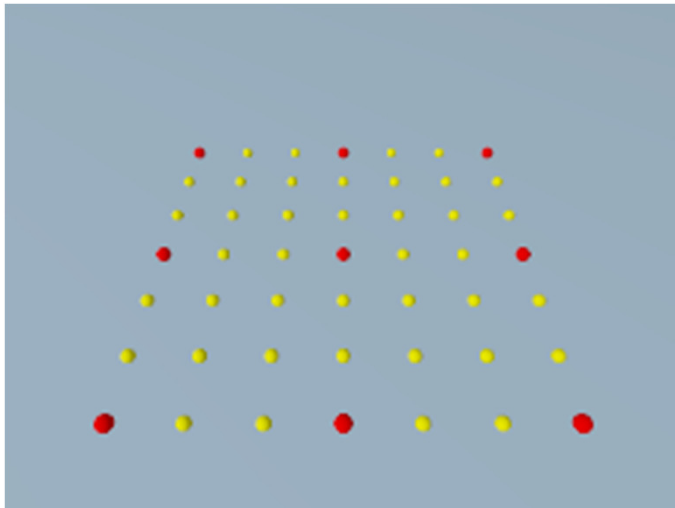
- ▶ Uniform tessellation is most straightforward
  - ▶ Evaluate points on a grid of  $u, v$  coordinates
  - ▶ Compute tangents at each point, take cross product to get per-vertex normal
  - ▶ Draw triangle strips with `glBegin(GL_TRIANGLE_STRIP)`



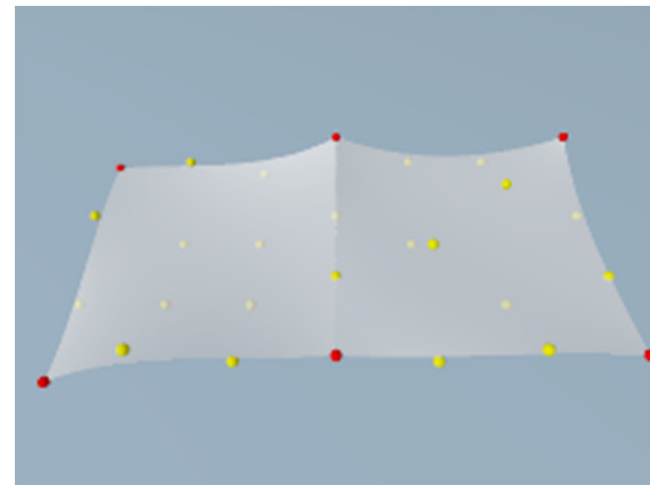
- ▶ Adaptive tessellation/recursive subdivision
  - ▶ Potential for “cracks” if patches on opposite sides of an edge divide differently
  - ▶ Tricky to get right, but can be done

# Piecewise Bézier Surface

- ▶ Lay out grid of adjacent meshes of control points
- ▶ For  $C^0$  continuity, must share points on the edge
  - ▶ Each edge of a Bézier patch is a Bézier curve based only on the edge mesh points
  - ▶ So if adjacent meshes share edge points, the patches will line up exactly
- ▶ But we have a crease...



Grid of control points

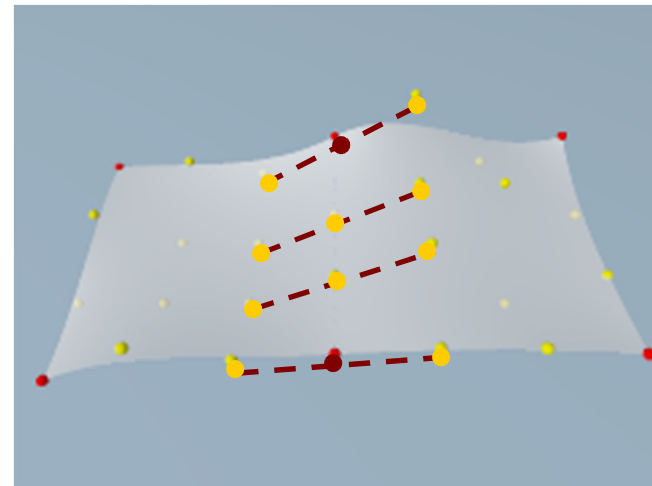
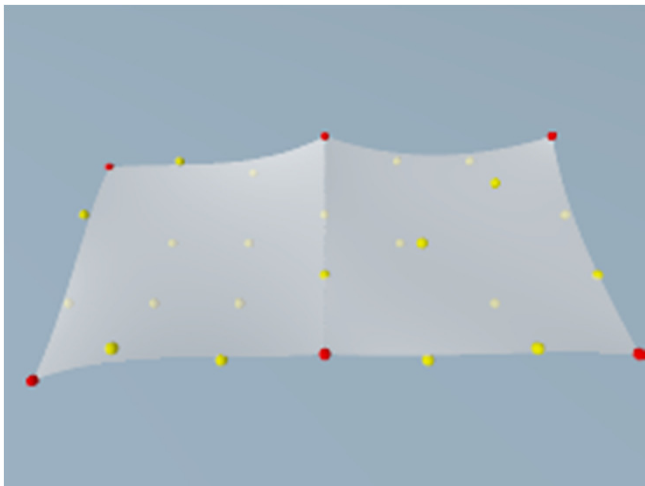


Piecewise Bézier surface

# $C^1$ Continuity

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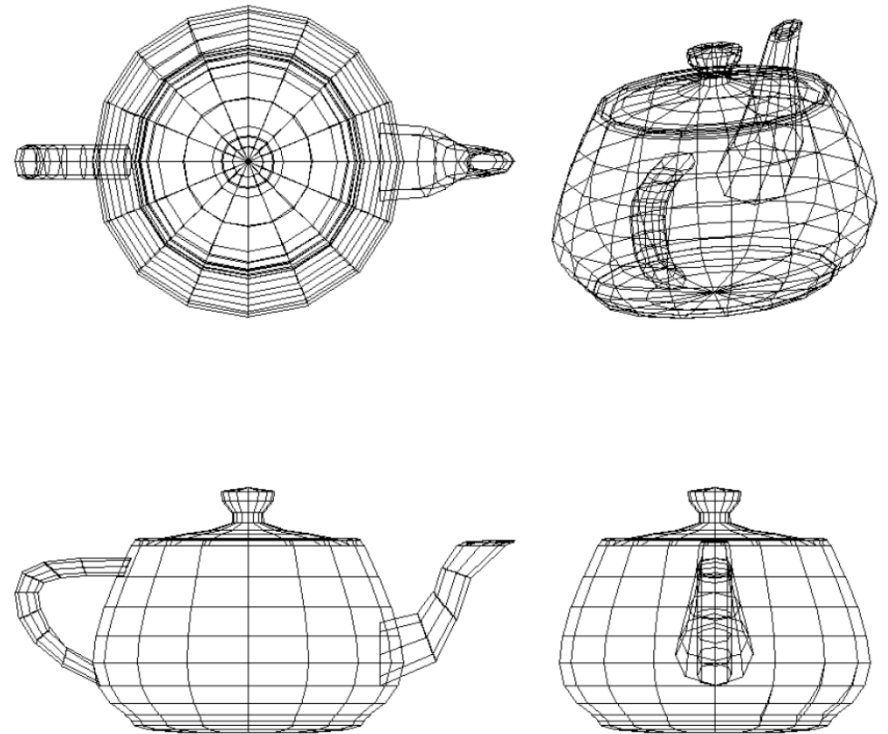
- ▶ We want the parametric curves that cross each edge to have  $C^1$  continuity
  - ▶ So the handles must be equal-and-opposite across the edge:



<http://www.spiritone.com/~english/cyclopedia/patches.html>

# Modeling With Bézier Patches

- ▶ Original Utah teapot, from Martin Newell's PhD thesis, consisted of 28 Bézier patches.
- ▶ The original had no rim for the lid and no bottom
- ▶ Later, four more patches were added to create a bottom, bringing the total to 32
- ▶ The data set was used by a number of people, including graphics guru Jim Blinn. In a demonstration of a system of his he scaled the teapot by .75, creating a stubbier teapot. He found it more pleasing to the eye, and it was this scaled version that became the highly popular dataset used today.



# Overview

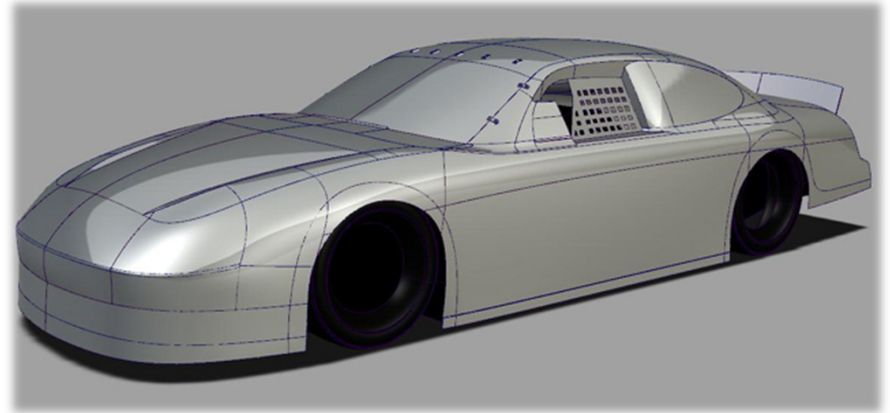
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- ▶ Bi-linear patch
- ▶ Bi-cubic Bézier patch
- ▶ Advanced parametric surfaces

# Problems with Bezier and NURBS Patches

- ▶ **NURBS surfaces are versatile**

- ▶ Conic sections
- ▶ Can blend, merge, trim...

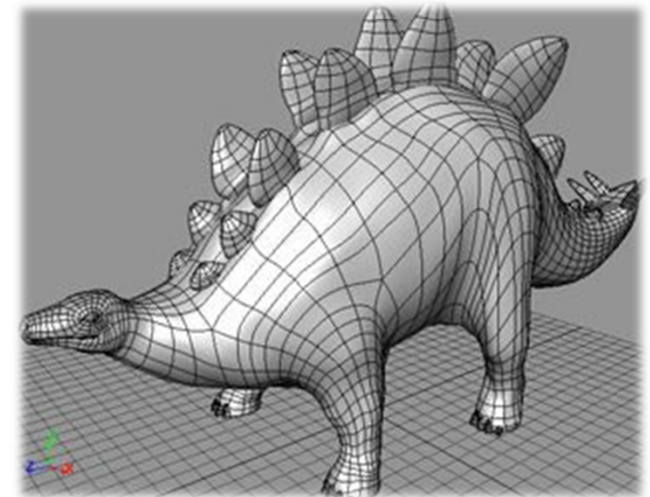


- ▶ **But:**

- ▶ Any surface will be made of quadrilateral patches (quadrilateral topology)

- ▶ **This makes it hard to**

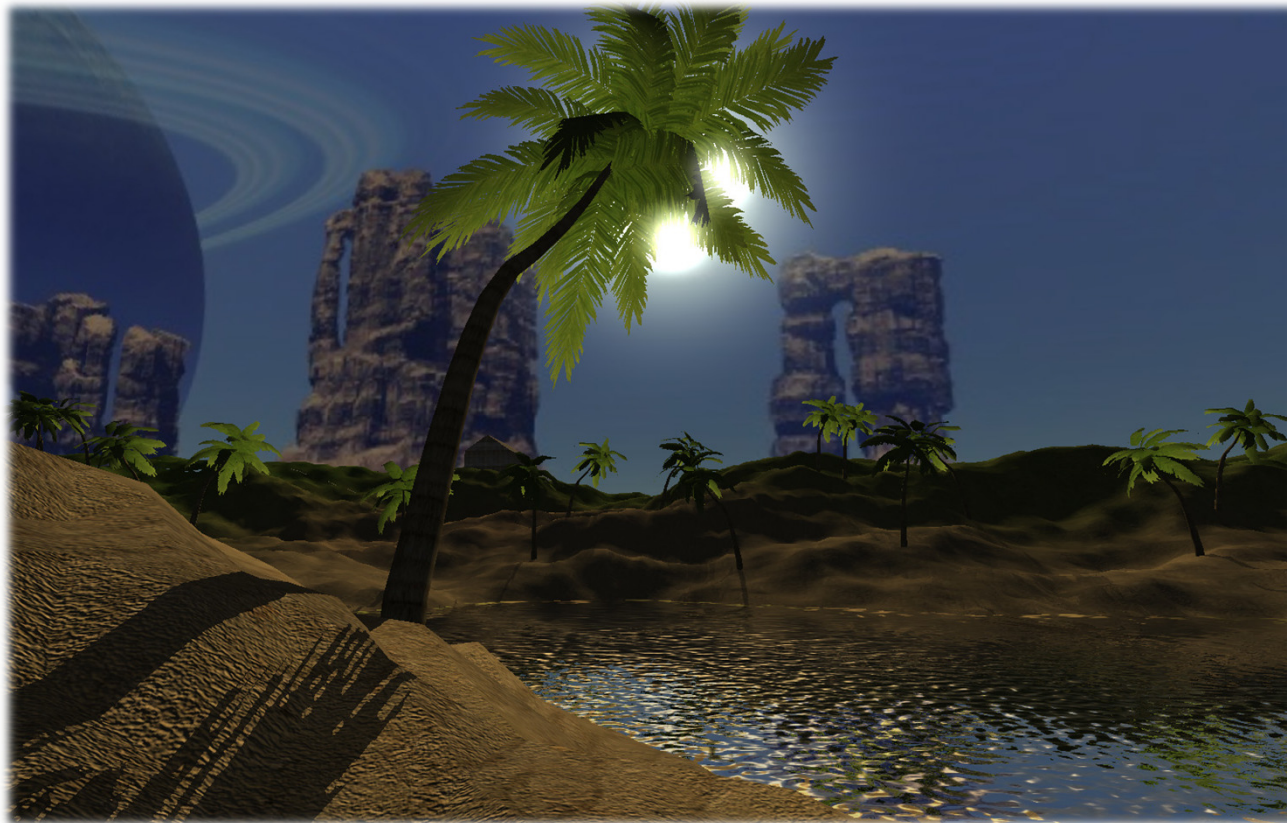
- ▶ Join or abut curved pieces
- ▶ Build surfaces with complex topology or structure



# GLSL

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- ▶ Real Time 3D Demo C++/OpenGL/GLSL Engine  
<http://www.youtube.com/watch?v=9N-kgCqy2xs>



# Lecture Overview

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- ▶ **Programmable Shaders**
  - ▶ Vertex Programs
  - ▶ Fragment Programs
  - ▶ GLSL

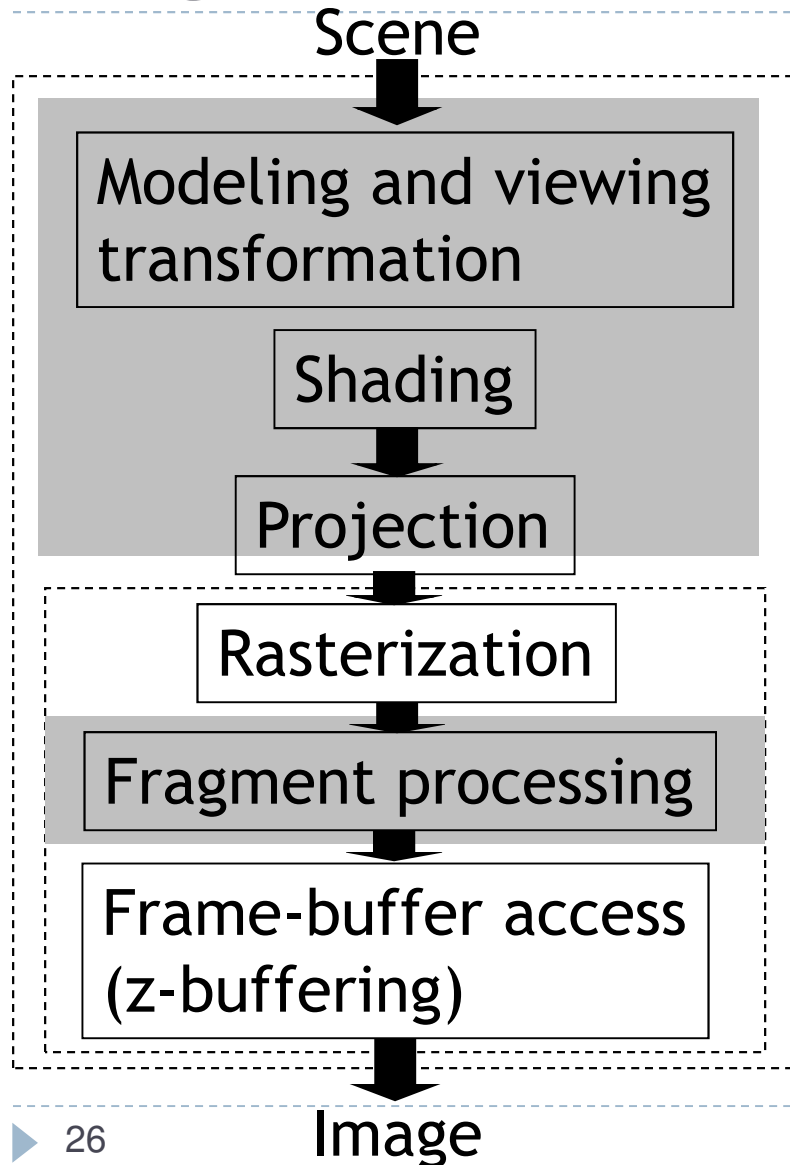


# Shader Programs

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- ▶ Programmable shaders consist of shader programs
- ▶ Written in a **shading language**
  - ▶ Syntax similar to C language
- ▶ Each shader is a separate piece of code in a separate ASCII text file
- ▶ Shader types:
  - ▶ Vertex shader
  - ▶ Tessellation shader
  - ▶ Geometry shader
  - ▶ Fragment shader (a.k.a. pixel shader)
- ▶ The programmer can provide any number of shader types to work together to achieve a certain effect
- ▶ If a shader type is not provided, OpenGL's fixed-function pipeline is used

# Programmable Pipeline



- ▶ Executed once per vertex:

- ▶ Vertex Shader
- ▶ Tessellation Shader
- ▶ Geometry Shader

- ▶ Executed once per fragment:

- ▶ Fragment Shader

# Vertex Shader

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- ▶ Executed once per vertex
- ▶ Cannot create or remove vertices
- ▶ Does not know the primitive it belongs to
- ▶ Replaces functionality for
  - ▶ Model-view, projection transformation
  - ▶ Per-vertex shading
- ▶ If you use a vertex program, you need to implement behavior for the above functionality in the program!
- ▶ Typically used for:
  - ▶ Character animation
  - ▶ Particle systems

# Tessellation Shader

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- ▶ Executed once per primitive
- ▶ Generates new primitives by subdividing each line, triangle or quad primitive
- ▶ Typically used for:
  - ▶ Adapting visual quality to the required level of detail
    - ▶ For instance, for automatic tessellation of Bezier curves and surfaces
  - ▶ Geometry compression: 3D models stored at coarser level of resolution, expanded at runtime
  - ▶ Allows detailed displacement maps for less detailed geometry

# Geometry Shader

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- ▶ Executed once per primitive (triangle, quad, etc.)
- ▶ Can create new graphics primitives from output of tessellation shader (e.g., points, lines, triangles)
  - ▶ Or can remove the primitive
- ▶ Typically used for:
  - ▶ Per-face normal computation
  - ▶ Easy wireframe rendering
  - ▶ Point sprite generation
  - ▶ Shadow volume extrusion
  - ▶ Single pass rendering to a cube map
  - ▶ Automatic mesh complexity modification (depending on resolution requirements)

# Fragment Shader

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- ▶ A.k.a. Pixel Shader
- ▶ Executed once per fragment
- ▶ Cannot access other pixels or vertices
  - ▶ Makes execution highly parallelizable
- ▶ Computes color, opacity, z-value, texture coordinates
- ▶ Typically used for:
  - ▶ Per-pixel shading (e.g., Phong shading)
  - ▶ Advanced texturing
  - ▶ Bump mapping
  - ▶ Shadows

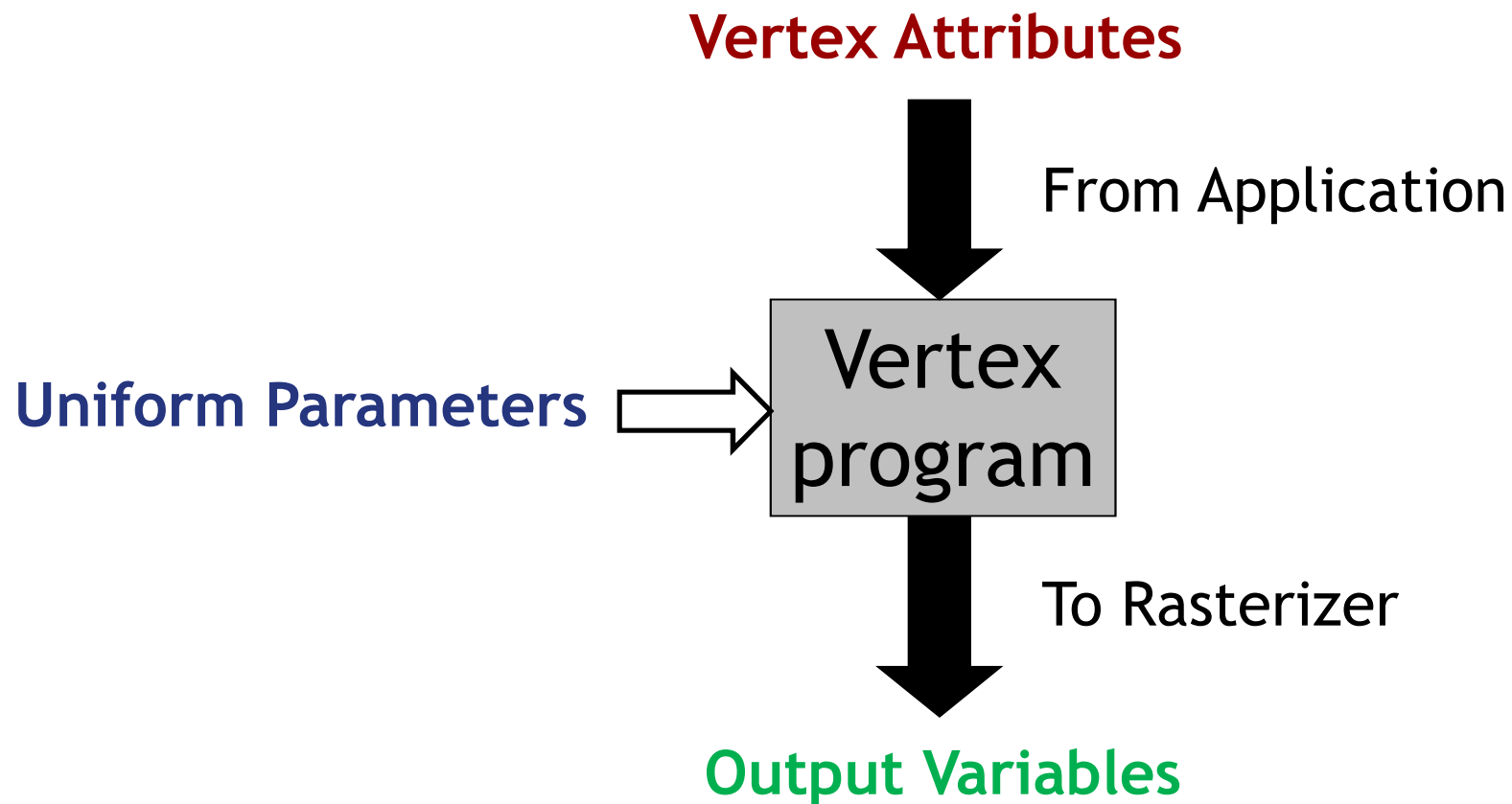
# Lecture Overview

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- ▶ Programmable Shaders
  - ▶ Vertex Programs
  - ▶ Fragment Programs
  - ▶ GLSL

# Vertex Programs

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# Vertex Attributes

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- ▶ Declared using the `attribute` storage classifier
- ▶ Different for each execution of the vertex program
- ▶ Can be modified by the vertex program
- ▶ Two types:
  - ▶ Pre-defined OpenGL attributes. Examples:  

```
attribute vec4 gl_Vertex;  
attribute vec3 gl_Normal;  
attribute vec4 gl_Color;
```
  - ▶ User-defined attributes. Example:  

```
attribute float myAttrib;
```

# Uniform Parameters

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- ▶ Declared by `uniform` storage classifier
- ▶ Normally the same for all vertices
- ▶ Read-only
- ▶ Two types:
  - ▶ Pre-defined OpenGL state variables
  - ▶ User-defined parameters

# Uniform Parameters: Pre-Defined

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- ▶ Provide access to the OpenGL state

- ▶ Examples for pre-defined variables:

```
uniform mat4 gl_ModelViewMatrix;  
uniform mat4 gl_ModelViewProjectionMatrix;  
uniform mat4 gl_ProjectionMatrix;  
uniform gl_LightSourceParameters  
gl_LightSource[gl_MaxLights];
```

# Uniform Parameters: User-Defined

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- ▶ Parameters that are set by the application
- ▶ Should not be changed frequently
  - ▶ Especially not on a per-vertex basis!
- ▶ **To access, use** `glGetUniformLocation`, `glUniform*` in application
- ▶ **Example:**
  - ▶ In shader declare  
`uniform float a;`
  - ▶ Set value of `a` in application:  
`GLuint p;`  
`int i = glGetUniformLocation(p, "a");`  
`glUniform1f(i, 1.0f);`

# Vertex Programs: Output Variables

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- ▶ Required output: homogeneous vertex coordinates

```
vec4 gl_Position
```

- ▶ **varying** output variables

- ▶ Mechanism to send data to the fragment shader

- ▶ Will be interpolated during rasterization

- ▶ Fragment shader gets interpolated data

- ▶ Pre-defined `varying` output variables, for example:

```
varying vec4 gl_FrontColor;
```

```
varying vec4 gl_TexCoord[];
```

Any pre-defined output variable that you do not overwrite will have the value of the OpenGL state.

- ▶ User-defined `varying` output variables, e.g.:

```
varying vec4 vertex_color;
```

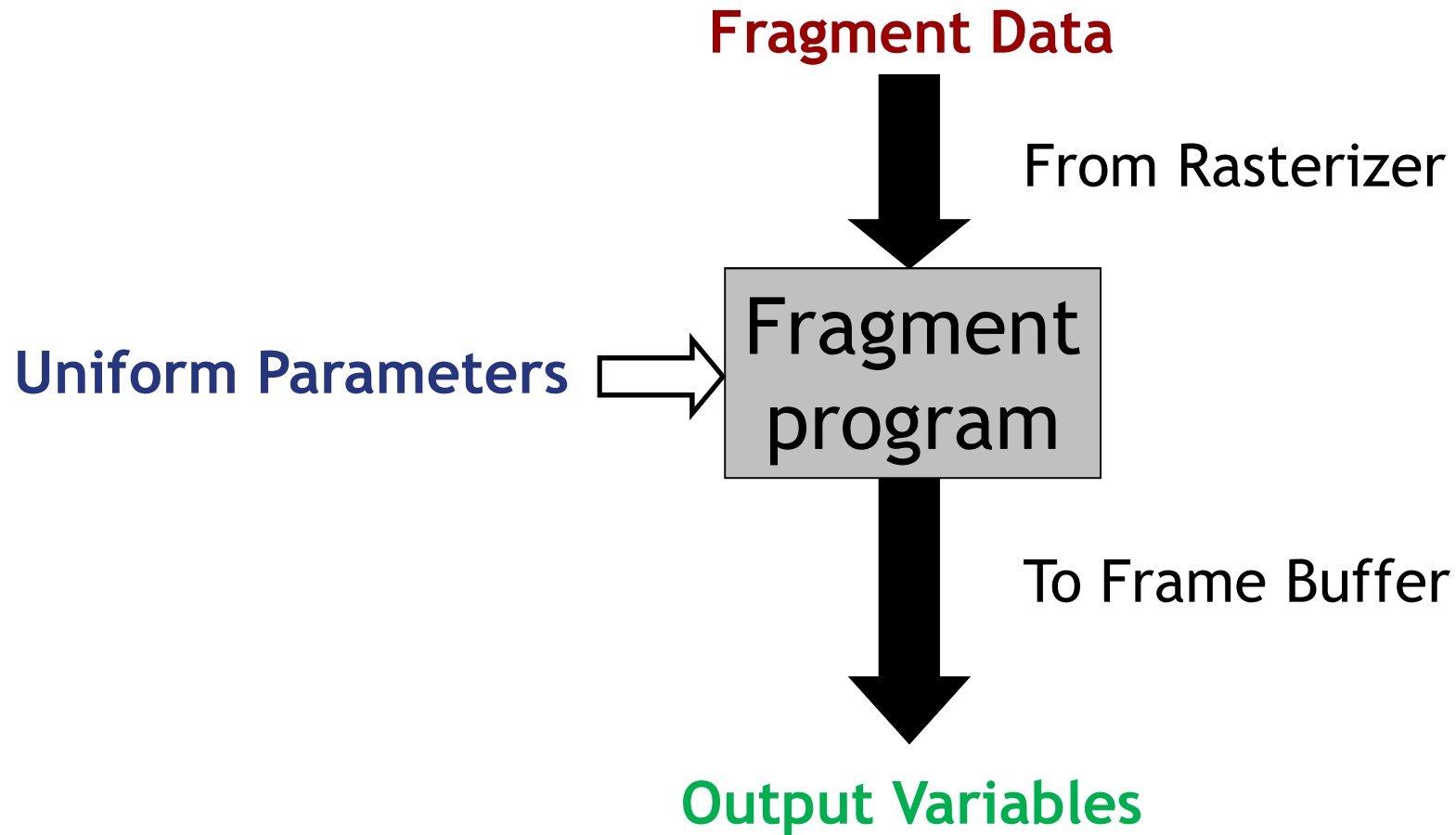
# Lecture Overview

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- ▶ Programmable Shaders
  - ▶ Vertex Programs
  - ▶ **Fragment Programs**
  - ▶ GLSL

# Fragment Programs

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# Fragment Data

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- ▶ Changes for each execution of the fragment program
- ▶ Fragment data includes:
  - ▶ Interpolated standard OpenGL variables for fragment shader, as generated by vertex shader, for example:  
`varying vec4 gl_Color;`  
`varying vec4 gl_TexCoord[];`
  - ▶ **Interpolated** `varying` **variables** from vertex shader
    - ▶ Allows data to be passed from vertex to fragment shader



# Uniform Parameters

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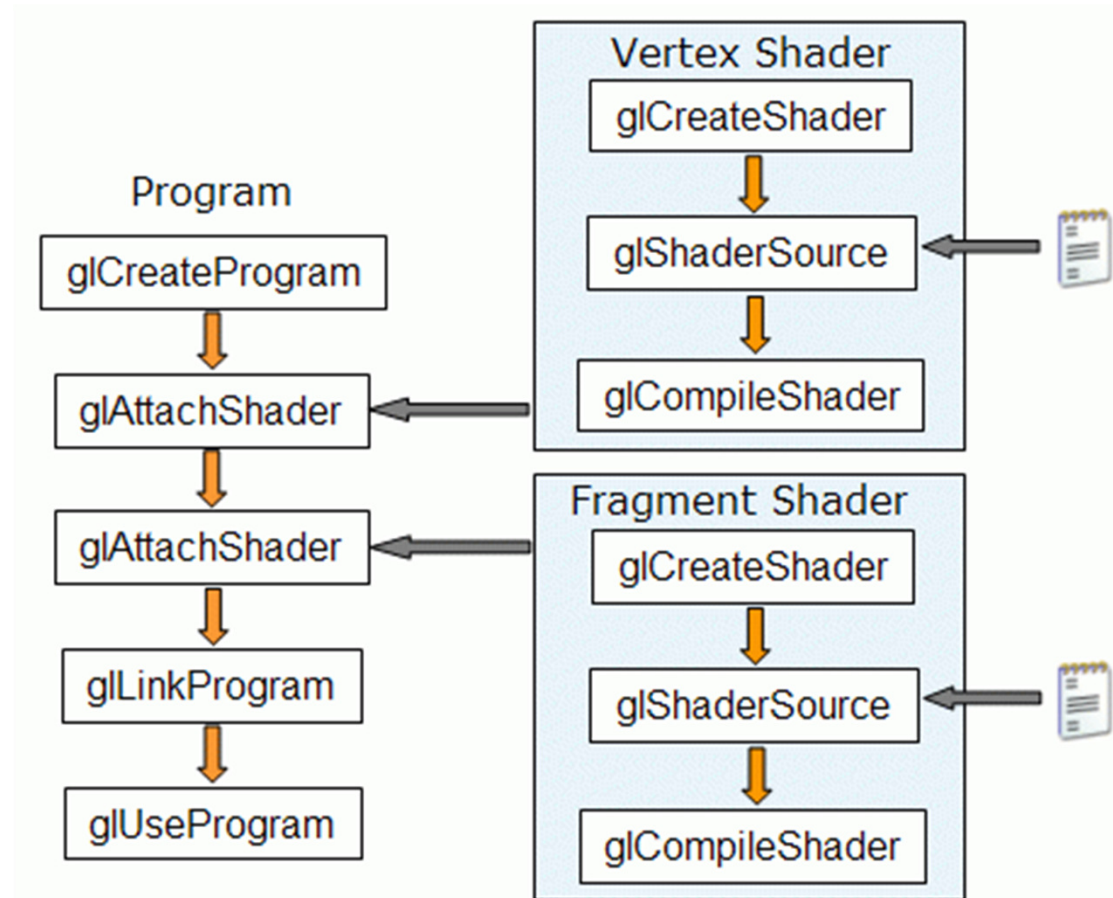
- ▶ Same as in vertex programs

# Output Variables

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- ▶ Pre-defined output variables:
  - ▶ `gl_FragColor`
  - ▶ `gl_FragDepth`
- ▶ OpenGL writes these to the frame buffer
- ▶ Result is undefined if you do not set these variables!

# Creating Shaders in OpenGL



Source: Gabriel Zachmann, Clausthal University

# Tutorials and Documentation

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- ▶ OpenGL and GLSL Specifications
  - ▶ <https://www.opengl.org/registry/>
- ▶ GLSL Tutorials
  - ▶ <http://www.lighthouse3d.com/opengl/glsl/>
  - ▶ <http://www.clockworkcoders.com/ogsl/tutorials.html>
- ▶ OpenGL Programming Guide (Red Book)
  - ▶ <http://www.glprogramming.com/red/>
- ▶ OpenGL Shading Language (Orange Book)
  - ▶ [http://wiki.labomedia.org/images/1/10/Orange\\_Book\\_-\\_OpenGL\\_Shading\\_Language\\_2nd\\_Edition.pdf](http://wiki.labomedia.org/images/1/10/Orange_Book_-_OpenGL_Shading_Language_2nd_Edition.pdf)
- ▶ OpenGL 4.5 API Reference Card
  - ▶ [https://www.opengl.org/sdk/docs/reference\\_card/opengl45-reference-card.pdf](https://www.opengl.org/sdk/docs/reference_card/opengl45-reference-card.pdf)