Name:________________________________

Your answers must include all steps of your derivations, or points will be deducted.

This is closed book exam. You may not use electronic devices, notes, textbooks or other written materials.

Throughout the exam we use right-handed coordinate systems only.

Good luck!

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1. View Frustum Culling (10 Points)

For this question, we have a robot named Robert, who becomes visible only when he is completely outside the view frustum (i.e., no point of Robert's body is inside the view frustum). For parts a) and b) assume a standard perspective projection.

a) Assume that we have all coordinates in view space, and that Robert is a sphere. Briefly describe how to compute a test for whether Robert is visible, given standard parameters for a symmetrical viewing frustum (FOV, aspect, near, far). Exact equations are preferred but not required; describe the algorithm in sufficient detail that it could be implemented by somebody who knows the equations. (4 points)

b) Assume instead that we have all coordinates in canonical view volume coordinates, a.k.a. Normalized Device Coordinates (NDC). The view frustum is now a cube in this coordinate system, which simplifies part of our visibility testing. However, it adds a significant complication. If we now perform the same procedure as in part a) using the faces of the cube in NDC and a sphere, our calculations will be off. Explain what the problem is. (3 points)

c) Assume that instead of perspective projection, we want to use a simple orthographic projection: we simply drop the view space z coordinate, and the view frustum is cubic. Does the problem described in part b) still occur? Why or why not? (3 points)
2. Backface Culling (10 Points)

a) What are the two main requirements for 3D models that allow them to be rendered with backface culling? (2 points)

b) If an object consists of 10,000 triangles, how many can approximately be culled with backface culling? (2 points)

c) Explain how backface culling works, using the example of the triangle below. Calculate the necessary vectors and show the math needed to decide if the triangle can be culled with backface culling or not. (6 points)
3. Environment mapping (10 Points)

a) What is environment mapping? (2 points)

b) How does its shading approach differ from illumination with the Phong Illumination Model? (2 points)

c) How could an environment map be captured? (2 points)

d) Is it easier to render perfectly specular objects (i.e., like a mirror), or diffuse objects, and why? (2 points)

e) Comparing spherical with cubic environment maps: name one advantage for each of them that it has over the other. (2 points)
4. Bezier curves (10 Points)

Suppose a Bézier curve \( C(u) \) is defined by the following four control points in the \( xy \)-plane: \( P_0 = (-2, 0), P_1 = (-2, 4), P_2 = (2, 4) \) and \( P_3 = (2, 0) \). Do the following problems:

a) What is the degree of \( C(u) \)? (1 point)

b) Geometrically compute \( C(0.5) \) with de Casteljau's algorithm: sketch the control points and add all additional points the de Casteljau algorithm produces, until you get \( C(0.5) \). What are the x/y coordinates for \( C(0.5) \)? (6 points)

c) Divide the curve at \( C(0.5) \), and list the control points of each curve segment in correct order. (3 points)
5. Surface Patches (10 Points)

Evaluating along a line requires an interpolation between two points. This concept can be extended to two dimensions creating a surface patch. Given the values for \( p_0 \), \( p_1 \), \( p_2 \), and \( p_3 \) below, find point \( x(\frac{1}{2}, \frac{1}{4}) \) following the steps below. \( x \) is defined as \( x(u, v) \).

\[
\begin{align*}
p_0 &= \langle -2, -4, 8 \rangle \\
p_1 &= \langle -6, 28, -24 \rangle \\
p_2 &= \langle 40, -14, -4 \rangle \\
p_3 &= \langle 32, -18, -12 \rangle
\end{align*}
\]

\[\text{Diagram illustrates the concept of a surface patch, but is not to scale.}\]

a) Find points \( q_0 \) and \( q_1 \) (6 points):

\[
\begin{align*}
q_0 &= \langle \quad \quad \quad \rangle \\
q_1 &= \langle \quad \quad \quad \rangle
\end{align*}
\]

b) Find point \( x \) (4 points):

\[
x = \langle \quad \quad \quad \rangle
6. Toon Shading (10 Points)

a) Why is toon shading also called Cel Shading? (2 points)

b) What are the two main visual effects that are used in toon shading to create the cartoon-style look? (4 points)

c) Explain how the toon shading algorithm detects silhouette edges. (4 points)
7. L-Systems (10 Points)

An L-system has the following parameters:

- Variables: F
- Constants: + -
- Start string: F
- Rule: \[ F \rightarrow F + F - F - F + F \]

F means “draw forward”, + means “turn left 90°”, and - means “turn right 90°”.

The initial drawing direction is to the right.

Hint: Level 0 of the recursion is: F

a) Generate the strings for levels one and two of the recursion (6 points).

b) Draw the curves for level 0, 1 and 2 of the recursion. (4 points)
8. Shadow mapping (10 Points)

a) Explain the difference between a hard shadow and a soft shadow. (2 points)

b) Explain what a shadow map is, how it is computed, and how it is used to determine whether an object is in the shadow of another object. (5 points)

c) Which of the following types of light does the shadow mapping algorithm work for and why? If it doesn't, why not? Point lights, directional lights, area lights. (3 points)