

CSE 167:  
Introduction to Computer Graphics  
Lecture #11: Performance Optimization

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# Announcements

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- ▶ Homework 5 due tomorrow at 1pm
- ▶ Homework 6 due next Friday

# Lecture Overview

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- ▶ Performance Optimization
  - ▶ Culling
  - ▶ Level of Detail Techniques

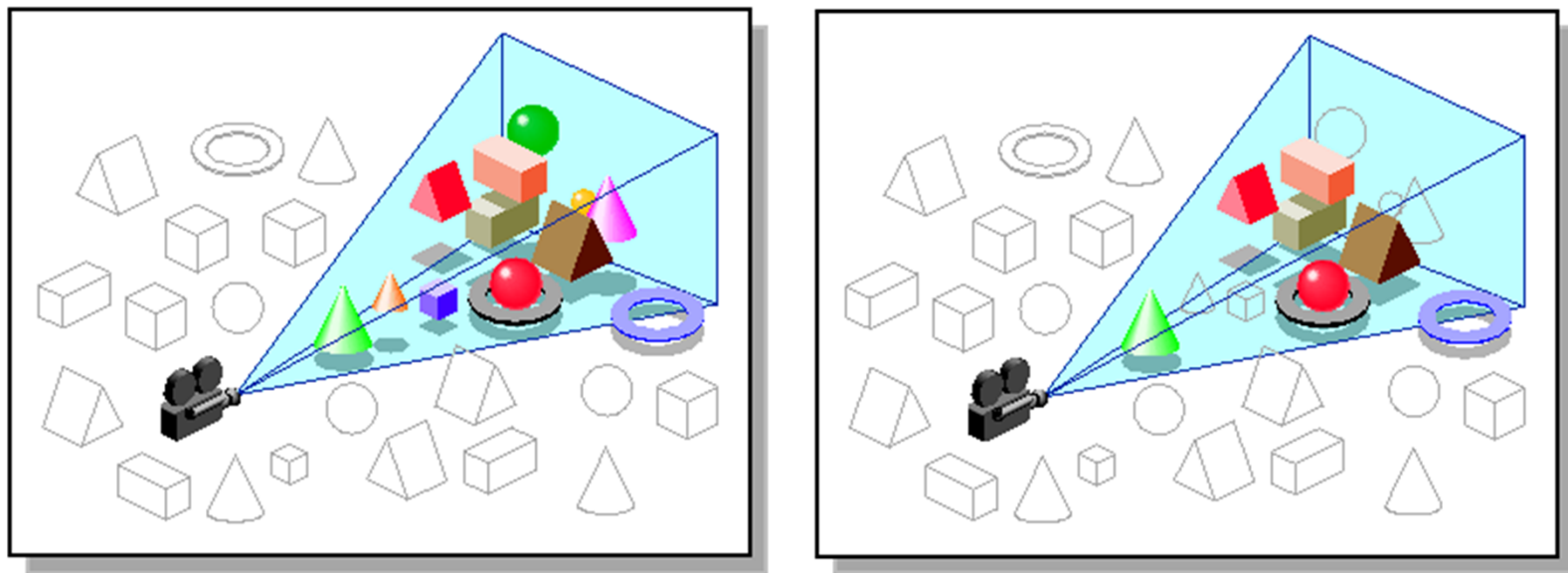
# Culling

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- ▶ Goal:  
Discard geometry that does not need to be drawn to speed up rendering
- ▶ Types of culling:
  - ▶ View frustum culling
  - ▶ Occlusion culling
  - ▶ Small object culling
  - ▶ Backface culling
  - ▶ Degenerate culling

# Occlusion Culling

- ▶ Geometry hidden behind occluder cannot be seen
  - ▶ Many complex algorithms exist to identify occluded geometry



*Images: SGI OpenGL Optimizer Programmer's Guide*

# Video

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- ▶ Umbra 3 Occlusion Culling explained
  - ▶ <http://www.youtube.com/watch?v=5h4QgDBwQhc>

# Small Object Culling

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- ▶ Object projects to less than a specified size
  - ▶ Cull objects whose screen-space bounding box is less than a threshold number of pixels

# Backface Culling

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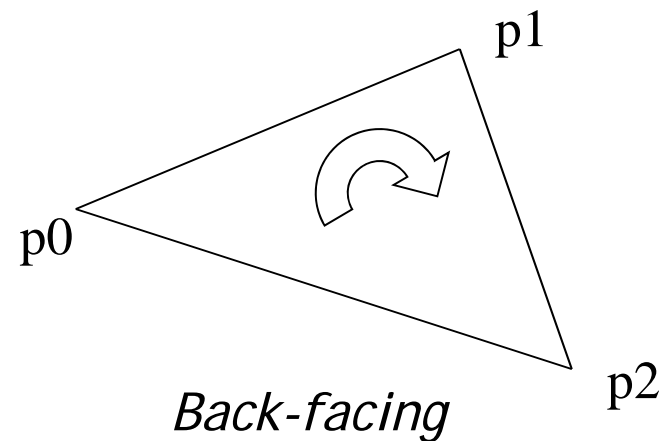
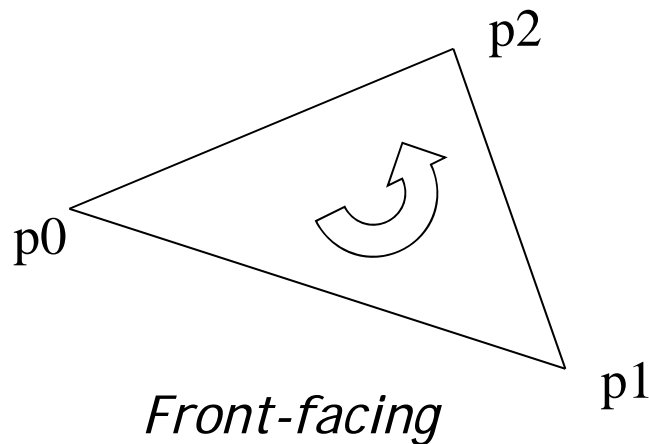
- ▶ Consider triangles as “one-sided”, i.e., only visible from the “front”
- ▶ Closed objects
  - ▶ If the “back” of the triangle is facing the camera, it is not visible
  - ▶ Gain efficiency by not drawing it (culling)
  - ▶ Roughly 50% of triangles in a scene are back facing



# Backface Culling

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- ▶ **Convention:**  
Triangle is front facing if vertices are ordered counterclockwise



- ▶ **OpenGL allows one- or two-sided triangles**
  - ▶ One-sided triangles:  
`glEnable(GL_CULL_FACE); glCullFace(GL_BACK)`
  - ▶ Two-sided triangles (no backface culling):  
`glDisable(GL_CULL_FACE)`

# Backface Culling

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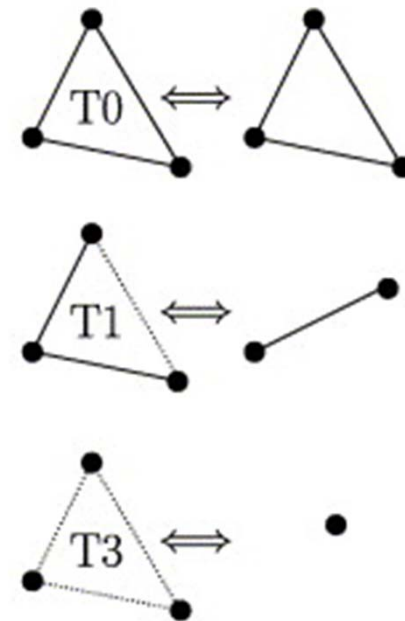
- ▶ Compute triangle normal after projection (homogeneous division)

$$\mathbf{n} = (\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{p}_2 - \mathbf{p}_0)$$

- ▶ Third component of  $\mathbf{n}$  negative: front-facing, otherwise back-facing
  - ▶ Remember: projection matrix is such that homogeneous division flips sign of third component

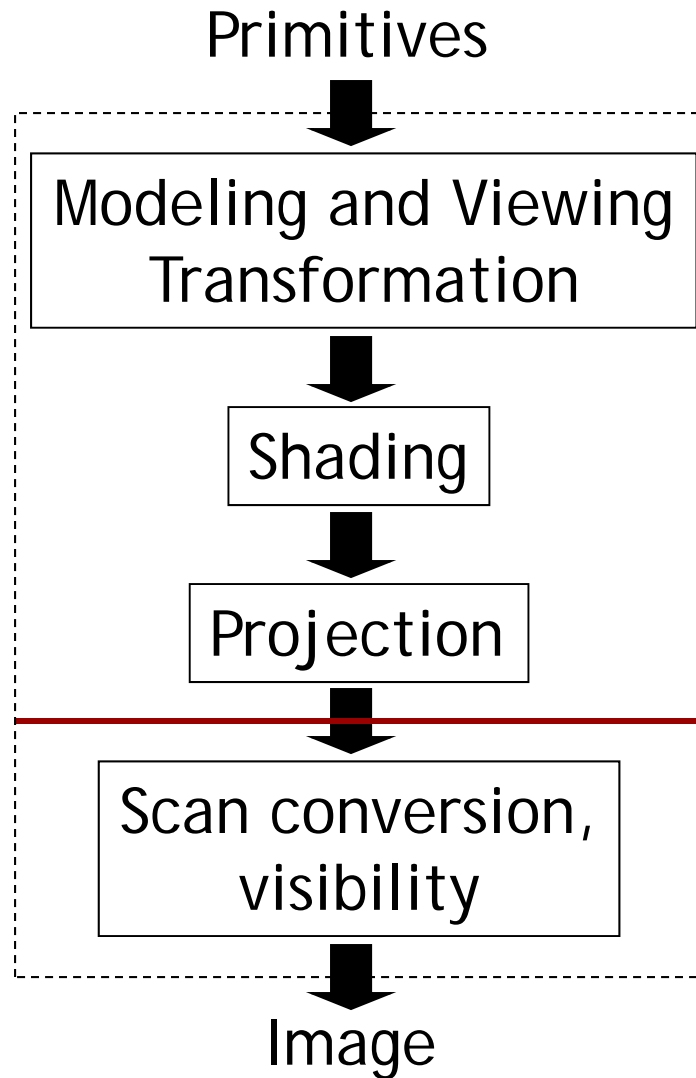
# Degenerate Culling

- ▶ Degenerate triangle has no area
  - ▶ Vertices lie in a straight line
  - ▶ Vertices at the exact same place
  - ▶ Normal  $\mathbf{n}=0$



Source: *Computer Methods in Applied Mechanics and Engineering*, Volume 194, Issues 48–49

# Rendering Pipeline



Culling, Clipping

- Discard geometry that will not be visible

# Level-of-Detail Techniques

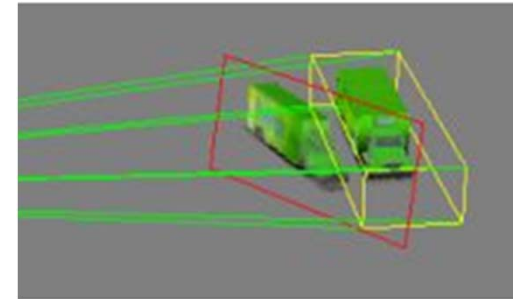
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- ▶ Don't draw objects smaller than a threshold

- ▶ Small feature culling
- ▶ Popping artifacts

- ▶ Replace 3D objects by 2D impostors

- ▶ Textured planes representing the objects



Impostor generation

- ▶ Adapt triangle count to projected size



Size dependent mesh reduction  
(Data: Stanford Armadillo)



Original vs. impostor

# Lecture Overview

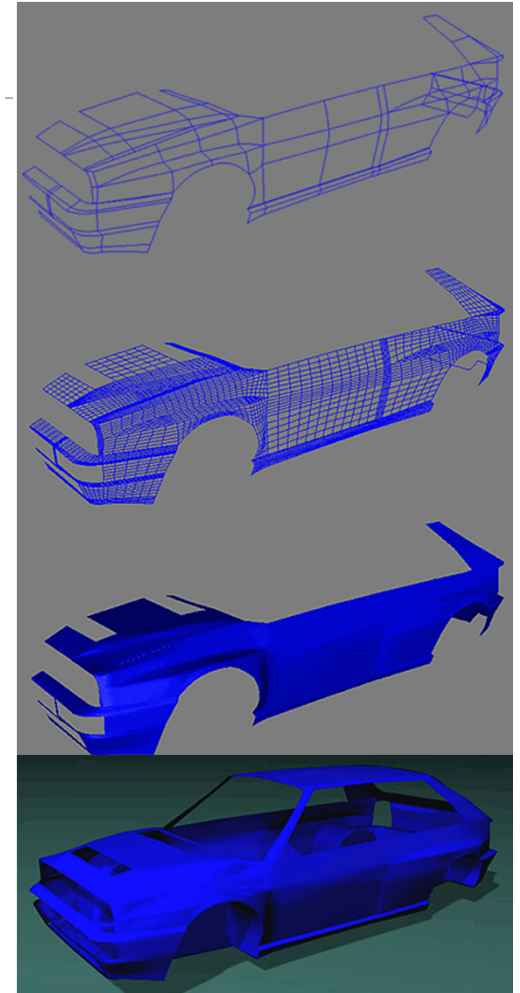
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- ▶ Polynomial Curves
  - ▶ Introduction
  - ▶ Polynomial functions
- ▶ Bézier Curves
  - ▶ Introduction
  - ▶ Drawing Bézier curves
  - ▶ Piecewise Bézier curves

# Modeling

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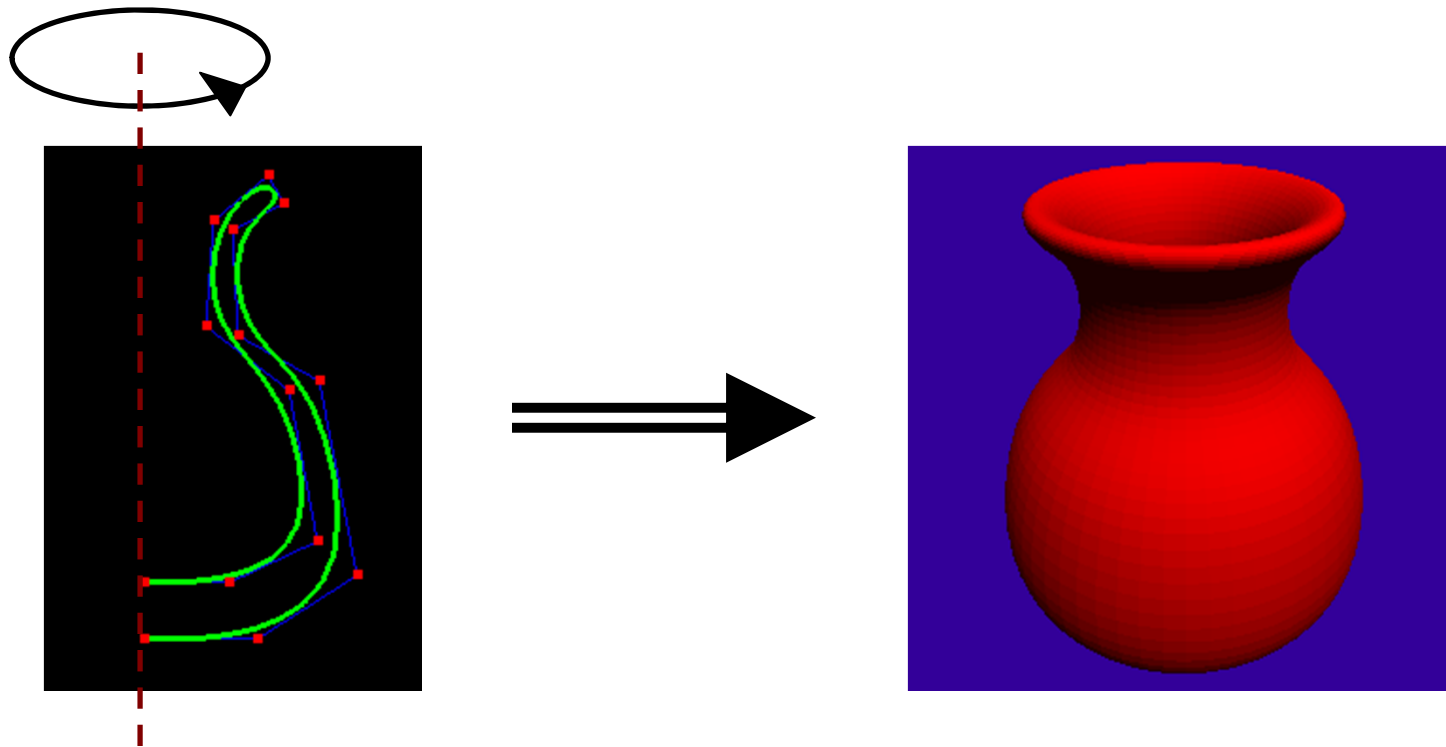
- ▶ Creating 3D objects
- ▶ How to construct complex surfaces?
- ▶ Goal
  - ▶ Specify objects with control points
  - ▶ Objects should be visually pleasing (smooth)
- ▶ Start with curves, then generalize to surfaces
- ▶ Next: What can curves be used for?



# Curves

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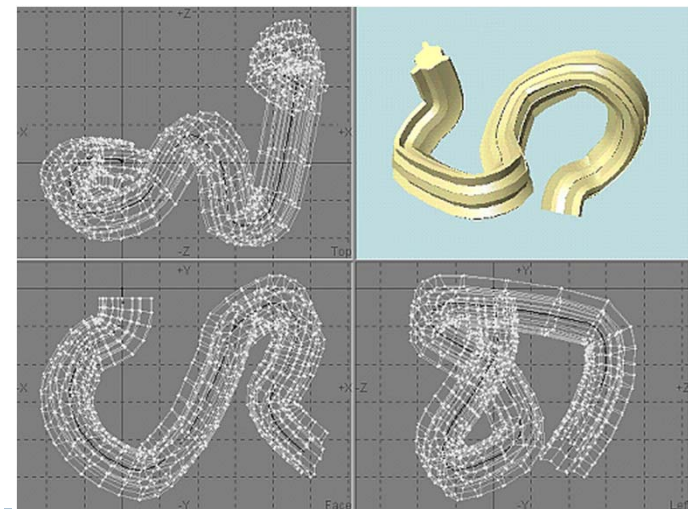
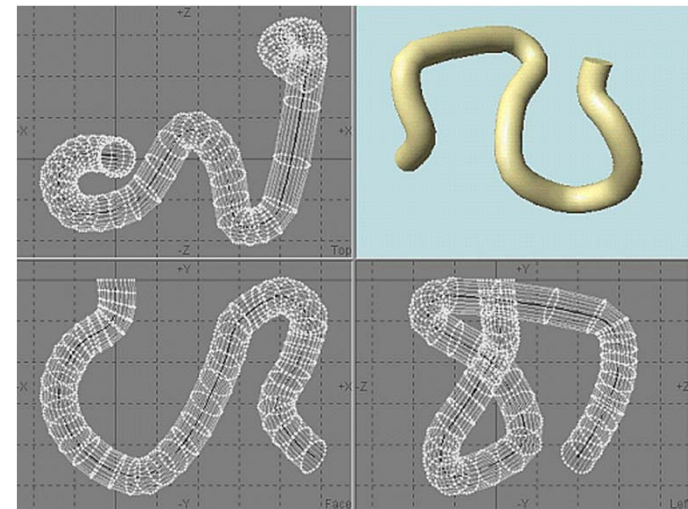
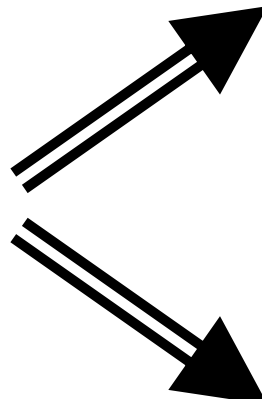
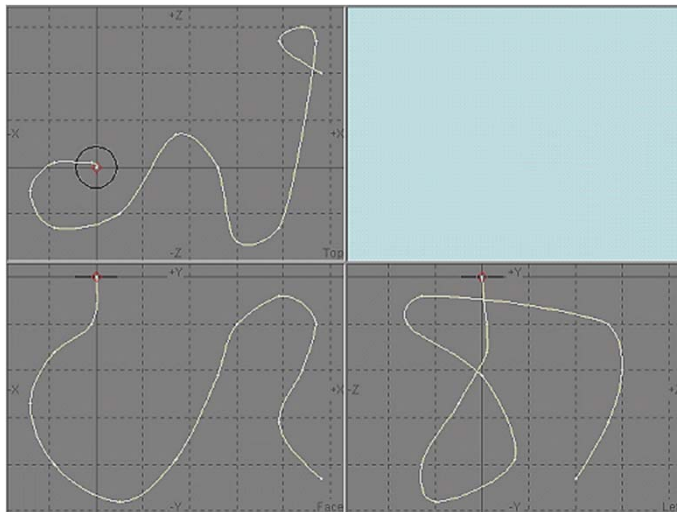
- ▶ Surface of revolution





# Curves

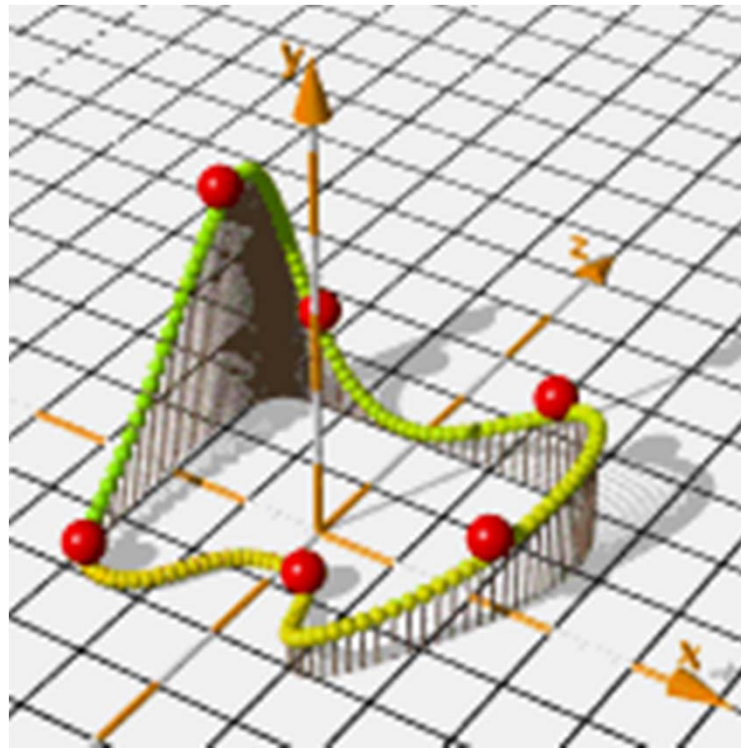
## ▶ Extruded/swept surfaces



# Curves

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- ▶ Animation
  - ▶ Provide a “track” for objects
  - ▶ Use as camera path

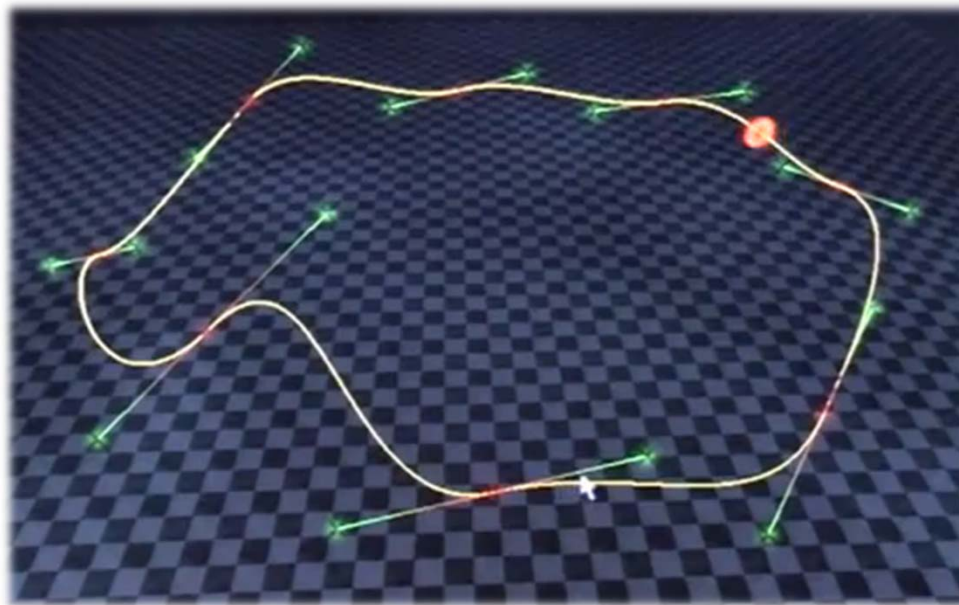


# Video

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- ▶ Bezier Curves

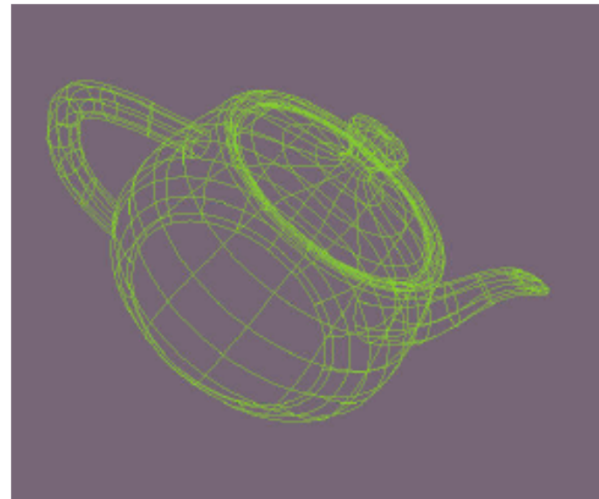
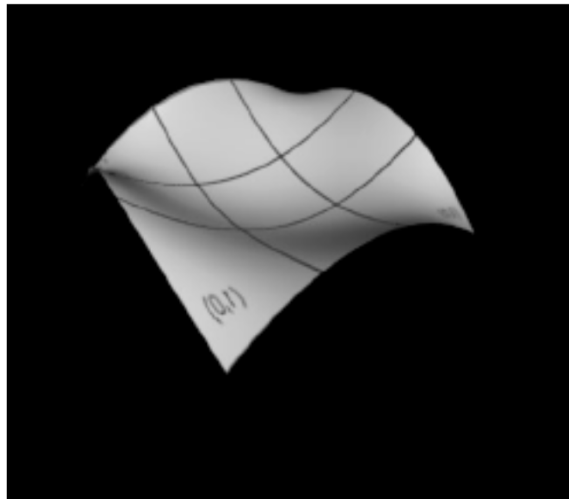
- ▶ <http://www.youtube.com/watch?v=hIDYJNEiYvU>



# Curves

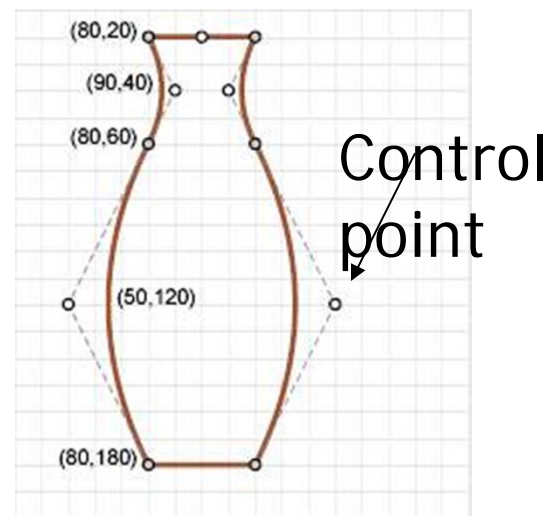
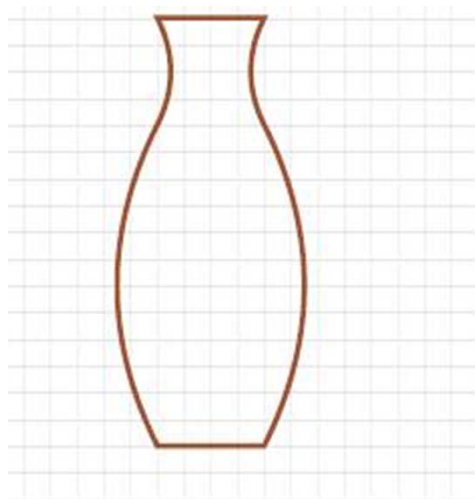
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- ▶ Can be generalized to surface patches



# Curve Representation

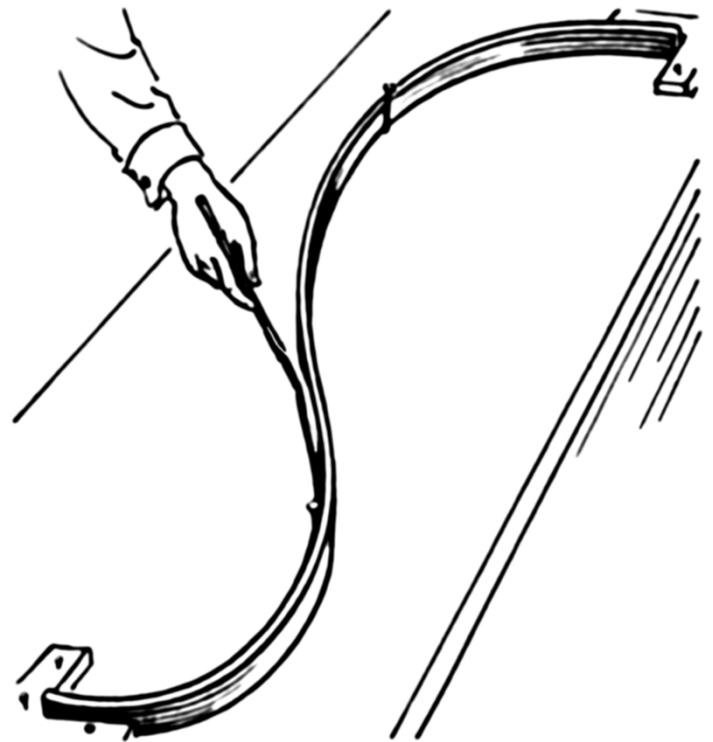
- ▶ Specify many points along a curve, connect with lines?
  - ▶ Difficult to get precise, smooth results across magnification levels
  - ▶ Large storage and CPU requirements
  - ▶ How many points are enough?
- ▶ Specify a curve using a small number of “control points”
  - ▶ Known as a *spline curve* or just *spline*



# Spline: Definition

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- ▶ **Wikipedia:**
  - ▶ Term comes from flexible spline devices used by shipbuilders and draftsmen to draw smooth shapes.
  - ▶ Spline consists of a long strip fixed in position at a number of points that relaxes to form a smooth curve passing through those points.



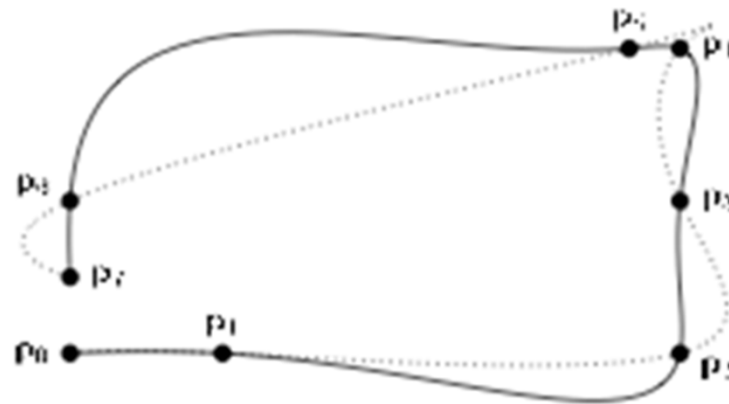
# Lecture Overview

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# Interpolating Control Points

- ▶ “Interpolating” means that curve goes through all control points
- ▶ Seems most intuitive
- ▶ Surprisingly, not usually the best choice
  - ▶ Hard to predict behavior
  - ▶ Hard to get aesthetically pleasing curves

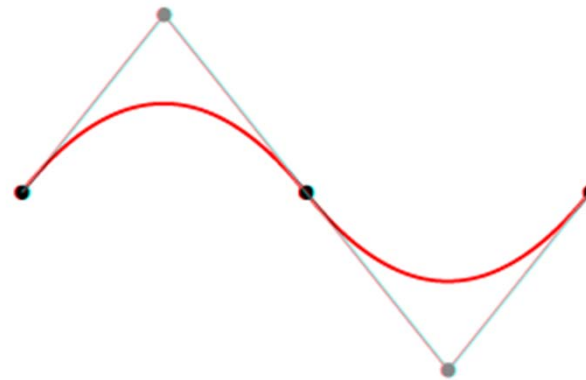




# Approximating Control Points

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- ▶ Curve is “influenced” by control points

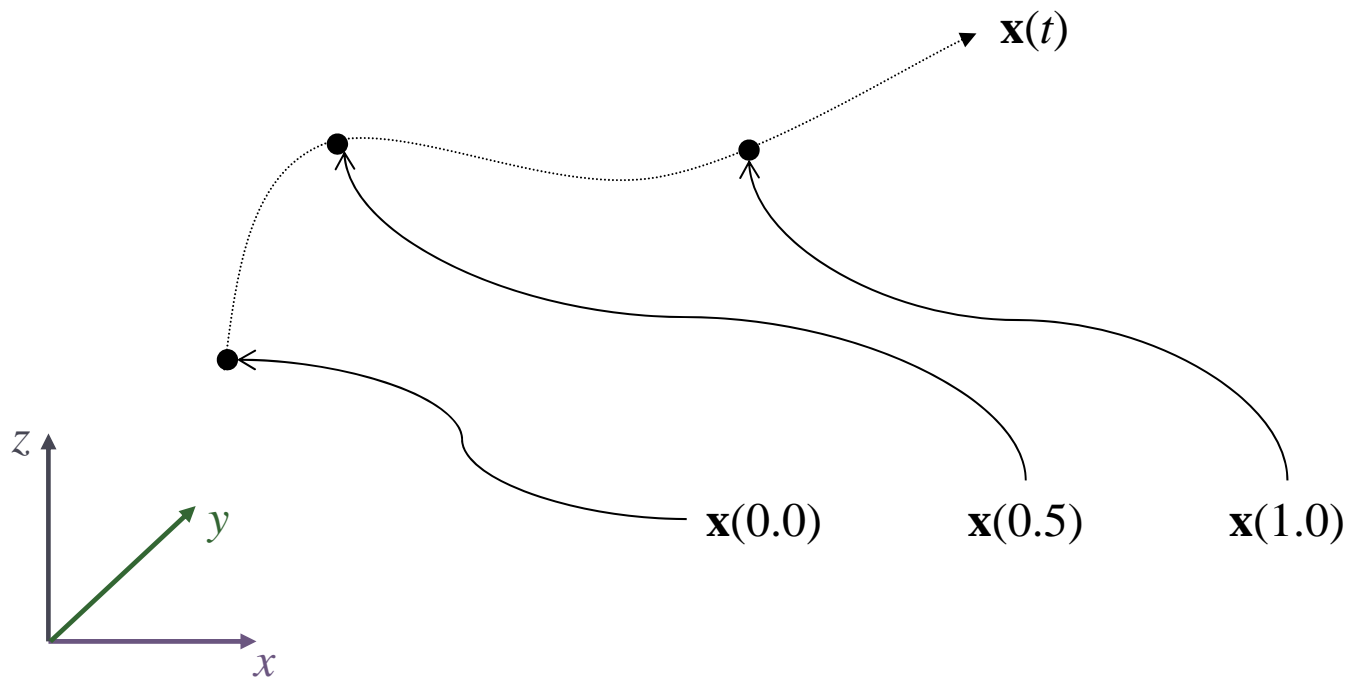


- ▶ Various types
- ▶ Most common: polynomial functions
  - ▶ Bézier spline (our focus)
  - ▶ B-spline (generalization of Bézier spline)
  - ▶ NURBS (Non Uniform Rational Basis Spline): used in CAD tools

# Mathematical Definition

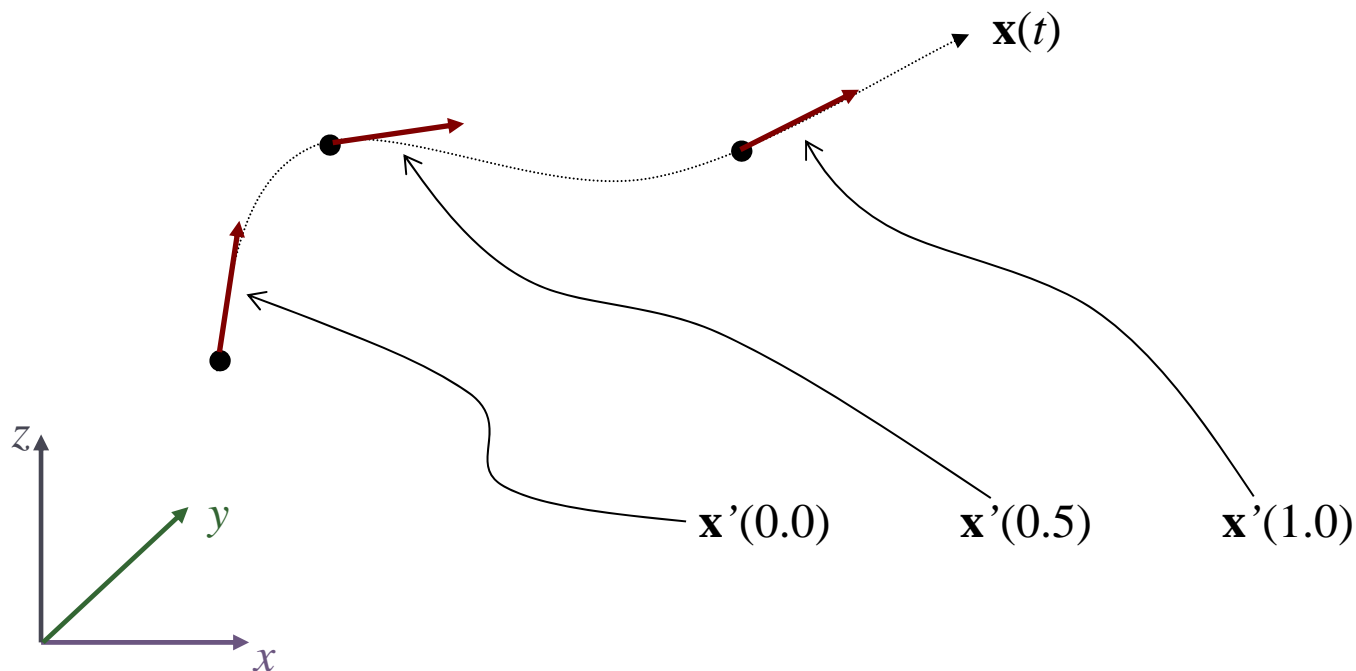
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- ▶ A vector valued function of one variable  $\mathbf{x}(t)$ 
  - ▶ Given  $t$ , compute a 3D point  $\mathbf{x}=(x,y,z)$
  - ▶ Could be interpreted as three functions:  $x(t)$ ,  $y(t)$ ,  $z(t)$
  - ▶ Parameter  $t$  “moves a point along the curve”



# Tangent Vector

- ▶ Derivative  $\mathbf{x}'(t) = \frac{d\mathbf{x}}{dt} = (x'(t), y'(t), z'(t))$
- ▶ Vector  $\mathbf{x}'$  points in direction of movement
- ▶ Length corresponds to speed



# Lecture Overview

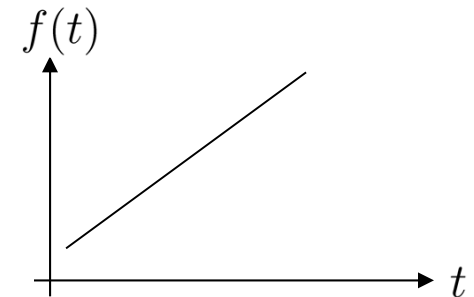
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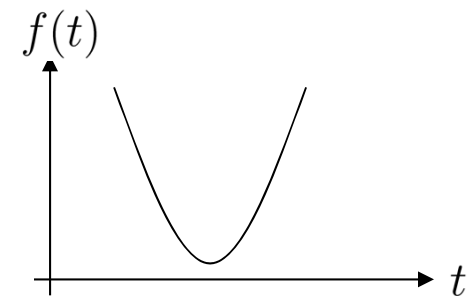
# Polynomial Functions

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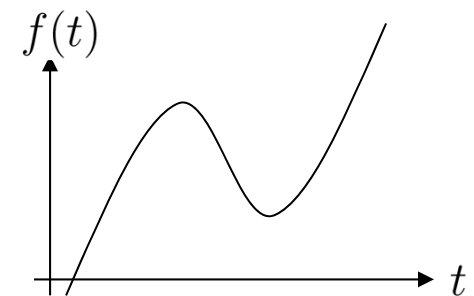
- ▶ **Linear:**  $f(t) = at + b$   
(1<sup>st</sup> order)



- ▶ **Quadratic:**  $f(t) = at^2 + bt + c$   
(2<sup>nd</sup> order)



- ▶ **Cubic:**  $f(t) = at^3 + bt^2 + ct + d$   
(3<sup>rd</sup> order)



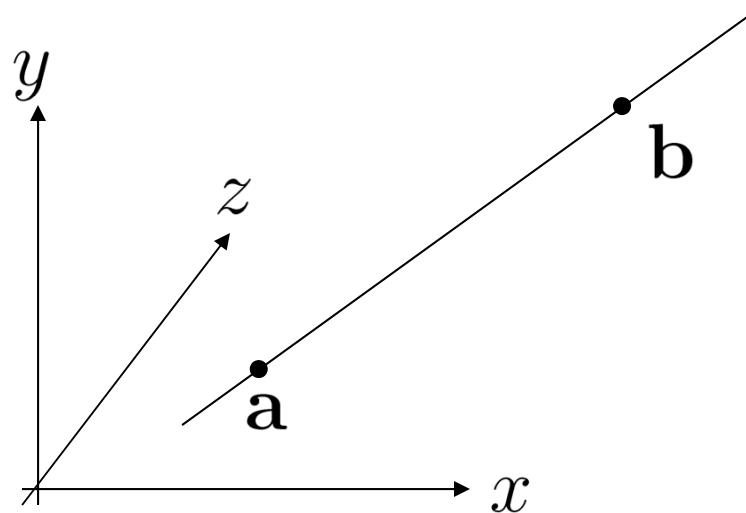
# Polynomial Curves

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- ▶ Linear  $\mathbf{x}(t) = \mathbf{a}t + \mathbf{b}$

$$\mathbf{x} = (x, y, z), \mathbf{a} = (a_x, a_y, a_z), \mathbf{b} = (b_x, b_y, b_z)$$

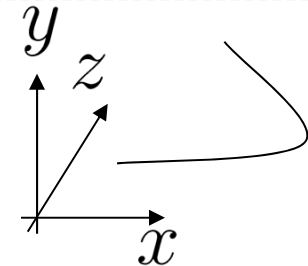
- ▶ Evaluated as:  
$$x(t) = a_x t + b_x$$
$$y(t) = a_y t + b_y$$
$$z(t) = a_z t + b_z$$



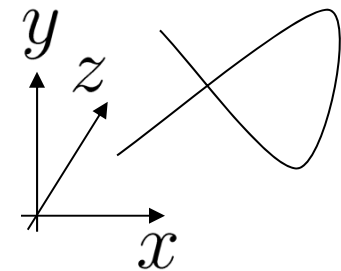
# Polynomial Curves

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► **Quadratic:**  $\mathbf{x}(t) = \mathbf{a}t^2 + \mathbf{b}t + \mathbf{c}$   
(2<sup>nd</sup> order)



► **Cubic:**  $\mathbf{x}(t) = \mathbf{a}t^3 + \mathbf{b}t^2 + \mathbf{c}t + \mathbf{d}$   
(3<sup>rd</sup> order)



► We usually define the curve for  $0 \leq t \leq 1$

# Control Points

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- ▶ Polynomial coefficients **a, b, c, d** can be interpreted as *control points*
  - ▶ Remember: **a, b, c, d** have  $x, y, z$  components each
- ▶ Unfortunately, they do not intuitively describe the shape of the curve
- ▶ Goal: intuitive control points



# Control Points

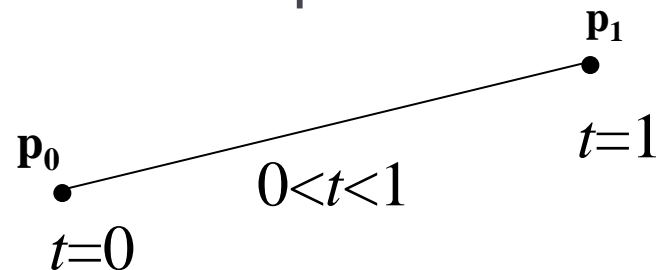
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- ▶ How many control points?
  - ▶ Two points define a line (1<sup>st</sup> order)
  - ▶ Three points define a quadratic curve (2<sup>nd</sup> order)
  - ▶ Four points define a cubic curve (3<sup>rd</sup> order)
  - ▶  $k+1$  points define a  $k$ -order curve
- ▶ Let's start with a line...

# First Order Curve

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- ▶ Based on linear interpolation (LERP)
  - ▶ Weighted average between two values
  - ▶ “Value” could be a number, vector, color, ...
- ▶ Interpolate between points  $\mathbf{p}_0$  and  $\mathbf{p}_1$  with parameter  $t$ 
  - ▶ Defines a “curve” that is straight (first-order spline)
  - ▶  $t=0$  corresponds to  $\mathbf{p}_0$
  - ▶  $t=1$  corresponds to  $\mathbf{p}_1$
  - ▶  $t=0.5$  corresponds to midpoint



$$\mathbf{x}(t) = \text{Lerp}(t, \mathbf{p}_0, \mathbf{p}_1) = (1 - t)\mathbf{p}_0 + t \mathbf{p}_1$$

# Linear Interpolation

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- ▶ Three equivalent ways to write it

- ▶ Expose different properties

1. Regroup for points  $\mathbf{p}$

$$\mathbf{x}(t) = \mathbf{p}_0(1 - t) + \mathbf{p}_1 t$$

2. Regroup for  $t$

$$\mathbf{x}(t) = (\mathbf{p}_1 - \mathbf{p}_0)t + \mathbf{p}_0$$

3. Matrix form

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{p}_0 & \mathbf{p}_1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1 \end{bmatrix}$$

# Weighted Average

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$$\mathbf{x}(t) = (1 - t)\mathbf{p}_0 + t\mathbf{p}_1$$
$$= B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1, \text{ where } B_0(t) = 1 - t \text{ and } B_1(t) = t$$

- ▶ Weights are a function of  $t$ 
  - ▶ Sum is always 1, for any value of  $t$
  - ▶ Also known as *blending functions*

