University of California San Diego
Department of Computer Science
CSE167: Introduction to Computer Graphics
Fall Quarter 2017
Midterm Examination #1
Tuesday, October 31st, 2017
Instructor: Dr. Jürgen P. Schulze

Name:________________________________

Your answers must include all steps of your derivations, or points will be deducted.

This is closed book exam. You may not use electronic devices, notes, textbooks or other written materials.

Good luck!

Do not write below this line

Exercise	**Max.**	**Points**
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	80	

1. Vector Properties (12 Points)

a) Given two vectors **a** and **b** in three-dimensional space, how do you calculate the angle between these vectors? (4 points)

b) If the vectors **a** and **b** were two sides of a triangle, how would you find the area of the triangle (employing the cross product)? (4 points)

c) How do two vectors **c** and **d** in 3D space have to be oriented with respect to one another to:

- maximize the dot product? (2 points)

- minimize the absolute value of the dot product ($|\boldsymbol{c} \cdot \boldsymbol{d}|$)? (2 points)

2. Transformations (10 Points)

Assume that you have a sequence of affine transformations to do on an object defined in model coordinates, with matrix **M** defining the transformation from model to world coordinates. Let's start with the individual transformations:

(a) Write **4 by 4** matrices for the following affine transformations in the model's own coordinate system:

1) Translation of the object by vector $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$. (2 points)

T =

2) Uniform scaling of the object by a factor of 2 about the origin of the model coordinate system (such that the object is two times the original size in all three dimensions). (1 point)

S =

3) Rotation of the object by 90 degrees (counterclockwise) around the z-axis. (2 points)

R =

(b) Develop the sequence of matrix multiplications, involving the transformation matrices from part (a), which results in matrix **A**, which combines the following operations in the given order. (5 points)

1) Translate the object with matrix **T** in the **world** coordinate system.

2) Rotate the object with matrix **R** about the origin of the **world** coordinate system.

3) Scale the object with matrix **S** about the origin of the **model** coordinate system.

4) Rotate the object with matrix **R** about the origin of the **model** coordinate system.

5) Translate the object with matrix **T** in the **world** coordinate system.

You don't need to calculate any numbers, just give the formula consisting of **R**, **S**, **T** and **M**.
Indicate inverse matrices with $^{-1}$. For example: T^{-1} is the inverse of T.
Assume that multiple concatenated matrix multiplications are executed from right to left (like on the course slides).

A =

3. Camera Coordinates (10 Points)

Given:

e =$[2\ \frac{1}{\sqrt{2}}\ \frac{-1}{\sqrt{2}}]^T$

d =$[1\ \frac{1}{\sqrt{2}}\ 1-\frac{1}{\sqrt{2}}]^T$

up =$[0\ 1\ 0]^T$

a) Using the above center of projection **e**, look at point **d**, and up vector **up**, calculate the basis vectors of the camera coordinate system they describe. (3 points)

b) Calculate the 4x4 camera matrix **C**. (1 point)

c) Calculate the inverse camera matrix $\mathbf{C}^{-1}$ (with derivation). (4 points)

d) Let **p**' = $\mathbf{C}^{-1}$ **p**. Given a point **p**' = (1, 2, 1) in camera coordinates, find **p** in world coordinates. (2 points)

4. Projection (10 Points)

A projection matrix collapses data from higher dimensions (typically three in graphics) to lower dimensions (typically two).

a) Which of the matrices below are projection matrices? Circle them. (5 points)

b) For each of the projection matrices indicate which dimension(s) collapse(s) to which dimension(s) in the projection, assuming a typical x, y, z coordinate system. (5 points)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{d} \end{bmatrix}$$

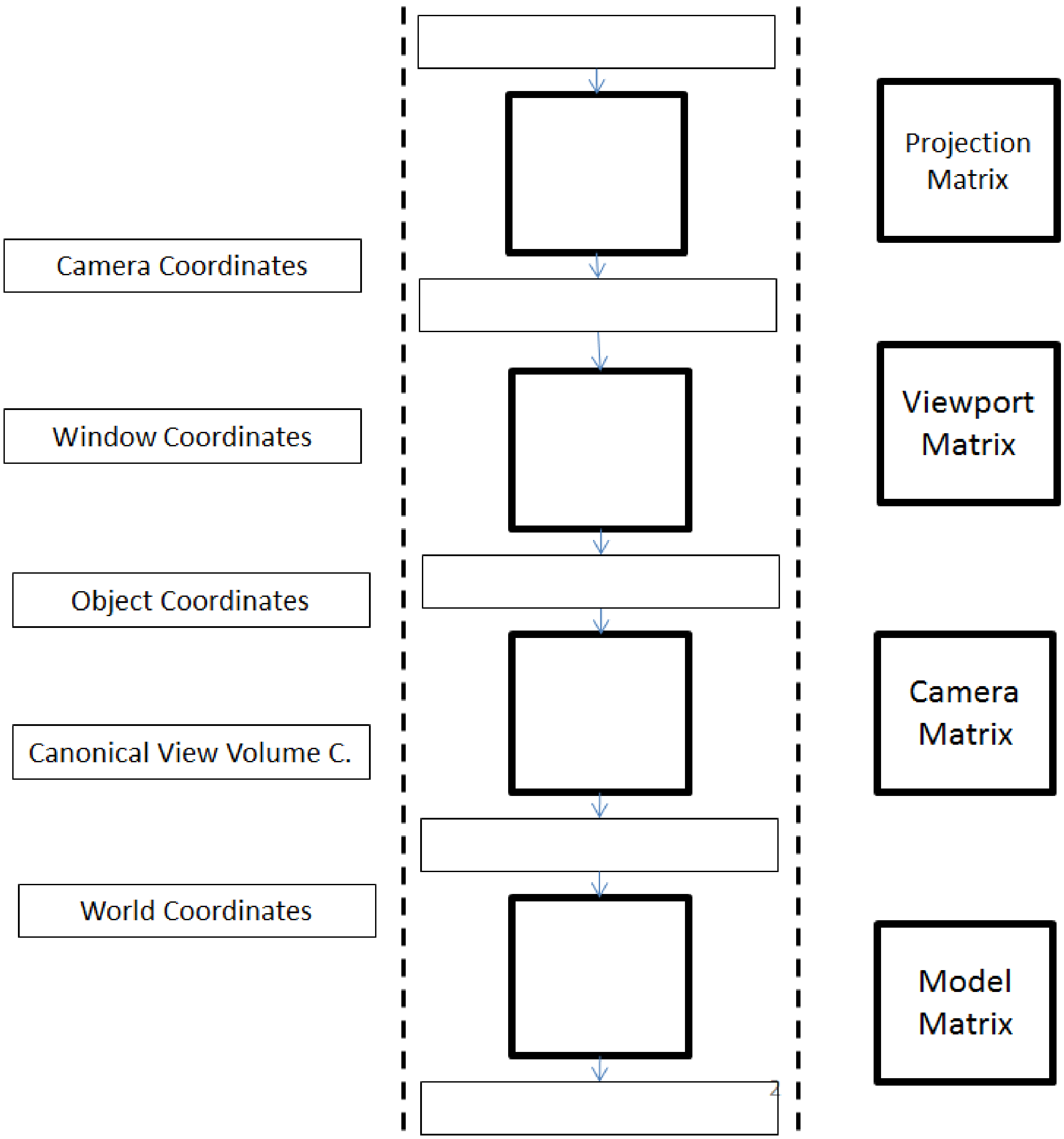
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Vertex Transformation (9 Points)

Fill in the Complete Vertex Transformation flow chart below (between the dashed lines) with the correct names for matrices (square boxes) and coordinate systems (flat boxes). You can save time by drawing arrows for where the respective elements (left and right of the dashed lines) go.



6. Illumination (10 Points)

Suppose that a shiny ground plane, $y = -2$, is illuminated by sunlight.

Let the sun be in direction $(1, 2, 3)$ and let the camera be at position
$(x, y, z) = (-6, 4, -4)$.

Determine the position on the ground plane at which the peak of the highlight occurs.

7. Lights (10 Points)

a) Name two differences between directional lights and point lights. (2 points)

b) Name the three parameters spot lights have which point lights don't have. (3 points)

c) How do the three distance attenuation options for point lights we covered in class differ from one another? (3 points)

d) Why are there multiple options for distance attenuation? (2 points)

8. Texture Mapping (10 Points)

A 300 × 200 texture image is mapped onto a 3D quad which is a parallelogram defined by vertices `p00`, `p10`, `p01`, `p11`.

Pixel (0, 0) in the texture maps to `p00`. Pixel (299, 0) maps to `p10`, etc.

What is the mapping from pixel (sp, tp) in the texture image to 3D coordinates on the quad? Write you answer as a product of matrices.